6

Availability and Irreversibility

6.1. AVAILABLE AND UNAVAILABLE ENERGY

There are many forms in which an energy can exist. But even under ideal conditions all these forms cannot be converted completely into work. This indicates that energy has two parts :

— *Available part.*

— *Unavailable part.*

'***Available energy***' is the *maximum portion of energy which could be converted into useful work by ideal processes which reduce the system to a dead state* (a state in equilibrium with the earth and its atmosphere). Because there can be only one value for maximum work which the system alone could do while descending to its dead state, it follows immediately that *'Available energy' is a property*.

A system which has a pressure difference from that of surroundings, work can be obtained from an expansion process, and if the system has a different temperature, heat can be transferred to a cycle and work can be obtained. But when the temperature and pressure becomes equal to that of the earth, transfer of energy ceases, and although the system contains internal energy, this energy is ***unavailable.***

Summarily available energy denote, the latent capability of energy to do work, and in this sense it can be applied to energy in the system or in the surroundings.

The theoretical maximum amount of work which can be obtained from a system at any state p_1 and T_1 when operating with a reservoir at the constant pressure and temperature p_0 and T_0 is called '**availability**'.

6.2. AVAILABLE ENERGY REFERRED TO A CYCLE

The ***available energy (A.E.)*** or the available part of the energy supplied is the maximum work output obtainable from a certain heat input in a cyclic heat engine (Fig. 6.1). The minimum energy that has to be rejected to the sink by the second law is called the ***unavailable energy (U.E.),*** or the unavailable part of the energy supplied.

$$\therefore \quad Q_1 = \text{A.E.} + \text{U.E.}$$

or $$W_{max} = \text{A.E.} = Q_1 - \text{U.E.}$$

For the given values of the source temperature T_1 and sink temperature T_2, the reversible efficiency,

$$\eta_{rev.} = 1 - \frac{T_2}{T_1}$$

For a given T_1, $\eta_{rev.}$ will increase with the decrease of T_2. *The lowest practicable temperature of heat rejection is the temperature of the surroundings,* T_0.

$$\therefore \qquad \eta_{max} = 1 - \frac{T_0}{T_1}$$

and
$$W_{max} = \left(1 - \frac{T_0}{T_1}\right) Q_1$$

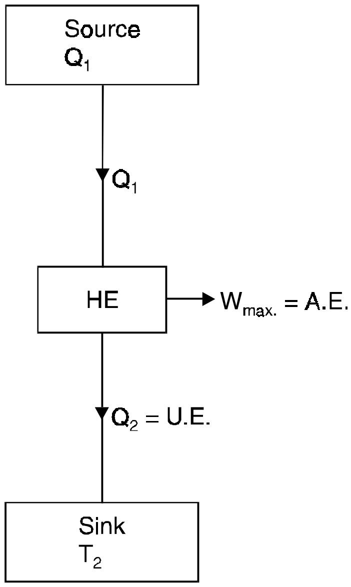


Fig. 6.1. Available and unavailable energy in a cycle.

Consider a finite process *l-m*, in which heat is supplied reversibly to a heat engine (Fig. 6.2). Taking an elementary cycle, if dQ_1 is the heat received by the engine reversibly at T_1,

Then
$$dW_{max} = \frac{T_1 - T_0}{T_1}\, dQ_1$$
$$= dQ_1 - \frac{T_0}{T_1} dQ_1 = \text{A.E.}$$

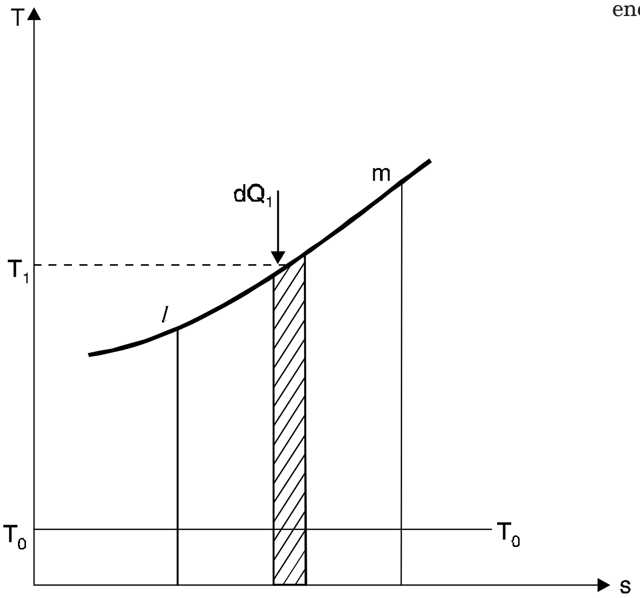


Fig. 6.2. Availability of energy.

For the heat engine receiving heat for the whole process *l-m*, and rejecting heat at T_0

$$\int_l^m dW_{max} = \int_l^m dQ_1 - \int_l^m \frac{T_0}{T_1}\, dQ_1$$

$$\therefore \qquad W_{max} = \text{A.E.} = Q_{l-m} - T_0\,(s_l - s_m) \qquad \text{...(6.1)}$$

or unavailable energy, $\quad \text{U.E.} = Q_{l-m} - W_{max}$

or $\quad \text{U.E.} = T_0\,(s_l - s_m)$

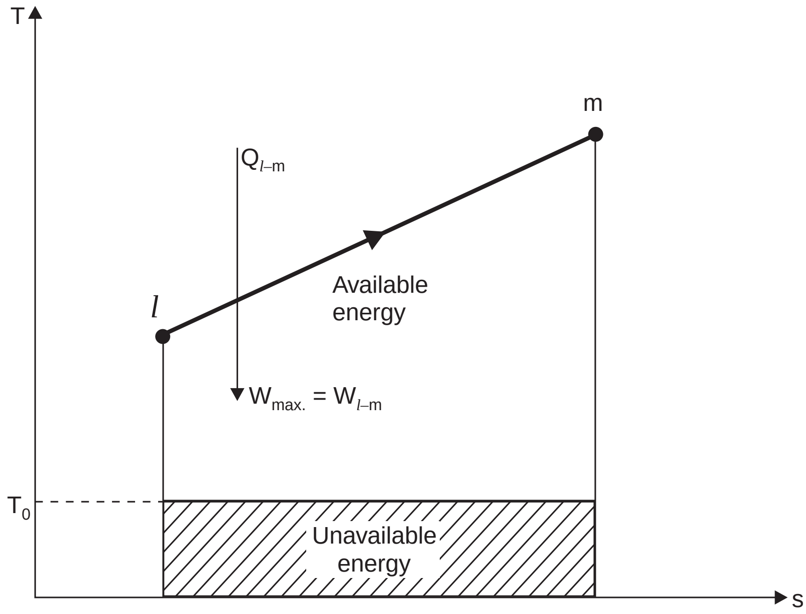


Fig. 6.3. Unavailable energy by the second law of thermodynamics.

Thus unavailable energy is the product of the lowest temperature of heat rejection, and the change of entropy of the system during the process of supplying heat (Fig. 6.3).

6.3. DECREASE IN AVAILABLE ENERGY WHEN HEAT IS TRANSFERRED THROUGH A FINITE TEMPERATURE DIFFERENCE

When transfer of heat takes place through a finite temperature difference, there is a decrease in the availability of energy so transferred. Consider a reversible heat engine operating between temperatures T_1 and T_0 (Fig. 6.4). Then

$$Q_1 = T \,.\, \Delta s\ ;$$

$$Q_2 = T_0\, \Delta s\ ;$$

and $$W = \text{A.E.} = [T_1 - T_0]\ \Delta s.$$

Assume that heat Q_1 is transferred through a finite temperature difference from the reservoir or source at T_1 to the engine absorbing heat at T_1', lower than T_1 (Fig. 6.4). The availability of Q_1 as received by the engine at T_1' can be found by allowing the engine to operate reversibly in a cycle between T_1' and T_0 receiving Q_1 and rejecting Q_2'.

Now, $$Q_1 = T_1 \Delta s = T_1' \Delta s'$$

$\because$ $$T_1 > T_1'$$

$\therefore$ $$\Delta s' > \Delta s$$

$$Q_2 = T_0\ \Delta s$$

$$Q_2' = T_0\ \Delta s'$$

$\because$ $$\Delta s' > \Delta s$$

$\therefore$ $$Q_2' > Q_2$$

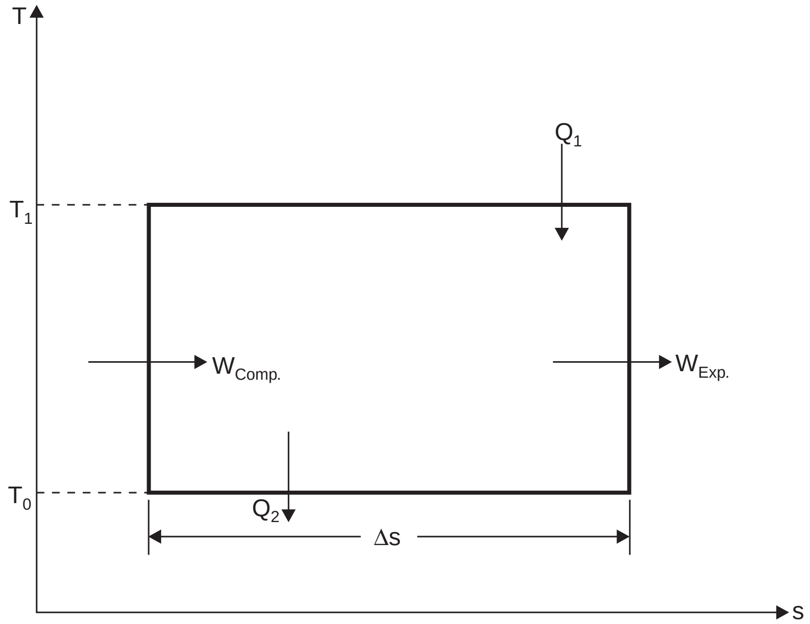


Fig. 6.4. Carnot-cycle.

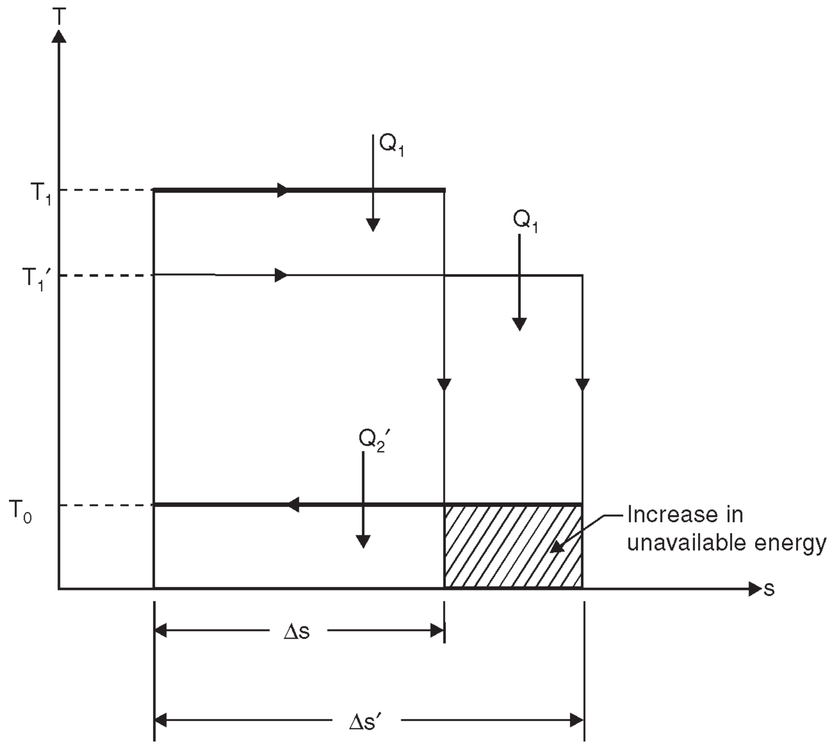


Fig. 6.5. Increase in unavailable energy due to heat transfer through a finite temperature difference.

$\therefore \qquad W' = Q_1 - Q_2' = T_1' \, \Delta s' - T_0 \, \Delta s'$

and $\qquad W = Q_1 - Q_2 = T_1 \, \Delta s - T_0 \, \Delta s$

$\therefore \qquad W' < W$, because $Q_2' > Q_2$

The *loss of available energy* due to irreversible heat transfer through finite temperature difference between the source and the working fluid during the heat addition process is given as :

$$W - W' = Q_2' - Q_2$$
$$= T_0 \, (\Delta s' - \Delta s)$$

i.e., Decrease in available energy, A.E.

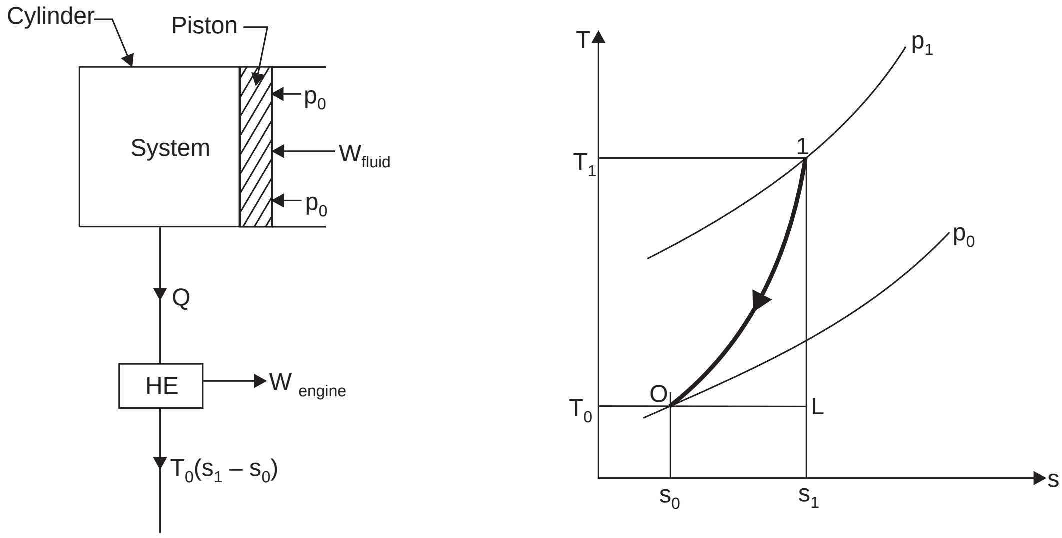
$$= T_0 \, (\Delta s' - \Delta s) \qquad \text{...(6.2)}$$

Thus the decrease in A.E. is the product of the lowest feasible temperature of heat rejection and the additional entropy change in the system while receiving heat irreversibly, compared to the case of reversible heat transfer from the same source. *The greater is the temperature difference* ($T_1 - T_1'$), *the greater is the heat rejection* Q_2' *and the greater will be the unavailable part of the energy supplied* (Fig. 6.5).

Energy is said to be *degraded* each time it flows through a finite temperature difference. That is, why the second law of thermodynamics is sometimes called the *law of the degradation of energy*, and energy is said to '*run down hill*'.

6.4. AVAILABILITY IN NON-FLOW SYSTEMS

Let us consider a system consisting of a fluid in a cylinder behind a piston, the fluid expanding reversibly from initial condition of p_1 and T_1 to final atmospheric conditions of p_0 and T_0. Imagine also that the system works in conjunction with a reversible heat engine which receives heat reversibly from the fluid in the cylinder such that the working substance of the heat engine follows the cycle *O1LO* as shown in Fig. 6.6, where $s_1 = s_L$ and $T_0 = T_L$ (the only possible way in which this could occur would be if an infinite number of reversible heat engines were arranged in parallel, each operating on a Carnot cycle, each one receiving heat at a different constant temperature and each one rejecting heat at T_0). The work done by the engine is given by :



(*a*) (*b*)

Fig. 6.6

W_{engine} = Heat supplied – Heat rejected

$$= Q - T_0 (s_1 - s_0) \quad ...(i)$$

The heat supplied to the engine is equal to the heat rejected by the fluid in the cylinder. Therefore, for the fluid in the cylinder undergoing the process 1 to 0, we have

$$-Q = (u_0 - u_1) + W_{fluid}$$

i.e.,

$$W_{fluid} = (u_1 - u_0) - Q \quad ...(ii)$$

Adding eqns. (*i*) and (*ii*), we get

$$W_{fluid} + W_{engine} = [(u_1 - u_0) - Q] + [Q - T_0 (s_1 - s_0)]$$
$$= (u_1 - u_0) - T_0 (s_1 - s_0)$$

The work done by the fluid on the piston is less than the total work done by the fluid, since there is no work done on the atmosphere which is at constant pressure p_0

i.e., Work done on atmosphere $= p_0 (v_0 - v_1)$

Hence, *maximum work available*

$$= (u_1 - u_0) - T_0 (s_1 - s_0) - p_0 (v_0 - v_1)$$

Note. When a fluid undergoes a complete cycle then the net work done on the atmosphere is zero.

$$W_{max} = (u_1 + p_0 v_1 - T_0 s_1) - (u_0 + p_0 v_0 - T_0 s_0) \quad ...(6.3)$$

$\therefore$

$$W_{max} = a_1 - a_0 \quad ...[6.3\ (a)]$$

The property, $a = u + p_0 v - T_0 s$ (per unit mass) is called the **non-flow availability function.**

6.5. AVAILABILITY IN STEADY FLOW SYSTEMS

Consider a fluid flowing steadily with a velocity C_1 from a reservoir in which the pressure and temperature remain constant at p_1 and T_1 through an apparatus to atmospheric pressure of p_0. Let the reservoir be at a height Z_1 from the datum, which can be taken at exit from the apparatus, *i.e.*, $Z_0 = 0$. For maximum work to be obtained from the apparatus the exit velocity, C_0, must be zero. It can be shown as for article 6.4 that a reversible heat engine working between the limits would reject $T_0 (s_1 - s_0)$ units of heat, where T_0 is the atmospheric temperature. Thus, we have

$$W_{max} = \left(h_1 + \frac{C_1^2}{2} + Z_1 g\right) - h_0 - T_0 (s_1 - s_0)$$

In several thermodynamic systems the kinetic and potential energy terms are negligible

i.e.,

$$W_{max} = (h_1 - T_0 s_1) - (h_0 - T_0 s_0)$$
$$= b - b_0$$

The property, $b = h - T_0 s$ (per unit mass) is called the **steady-flow availability function.**

[In the equation $b = h - T_0 s$; the function '*b*' (like the function '*a*') is a composite property of a system and its environment ; this is also known as ***Keenan function***].

Note 1. The alternative names for availability and unavailable quantity $T_0 \Delta s$ are ***energy*** and a ***energy*** respectively.

2. The only difference between $a = u + p_0 v - T_0 s$ function and $b = (h - T_0 s) = (u + pv - T_0 s)$ function is in pressure only.

6.6. HELMHOLTZ AND GIBBS FUNCTIONS

The work done in a non-flow reversible system (per unit mass) is given by :

$$W = Q - (u_0 - u_1)$$
$$= T.ds - (u_0 - u_1)$$

$$= T(s_0 - s_1) - (u_0 - u_1)$$

i.e., $$W = (u_1 - Ts_1) - (u_0 - Ts_0) \quad ...(6.4)$$

The term $(u - Ts)$ is known as *Helmholtz function*. This gives maximum possible output when the heat Q is transferred at constant temperature and is the case with a very large source.

If work against atmosphere is equal to $p_0 (v_0 - v_1)$, then the maximum work available,

$$W_{max} = W - \text{work against atmosphere}$$
$$= W - p_0 (v_0 - v_1)$$
$$= (u_1 - Ts_1) - (u_0 - Ts_0) - p_0 (v_0 - v_1)$$
$$= (u_1 + p_0v_1 - Ts_1) - (u_0 + p_0v_0 - Ts_0)$$
$$= (h_1 - Ts_1) - (h_0 - Ts_0)$$

i.e., $$W_{max} = g_1 - g_0 \quad ...(6.5)$$

where $g = h - T.s$ is known as ***Gibb's function*** **or** ***free energy function.***

The maximum possible available work when system changes from 1 to 2 is given by

$$W_{max} = (g_1 - g_0) - (g_2 - g_0) = g_1 - g_2 \quad ...(6.6)$$

Similarly, for *steady flow system* the maximum work available is given by

$$W_{max} = (g_1 - g_2) + (\text{KE}_1 - \text{KE}_2) + (\text{PE}_1 - \text{PE}_2) \quad ...(6.7)$$

where K.E. and P.E. represent the kinetic and potential energies.

It may be noted that Gibb's function $g = (h - Ts)$ is a property of the system where availability function $a = (u + p_0v - T_0s)$ is a composite property of the system and surroundings.

Again,
$$a = u + p_0v - T_0s$$
$$b = u + pv - T_0s$$
$$g = u + pv - Ts$$

When state 1 proceeds to *dead state* (zero state)

$$a = b = g.$$

6.7. IRREVERSIBILITY

The actual work which a system does is always *less* than the idealized reversible work, and the difference between the two is called the *irreversibility of the process*.

Thus, Irreversibility, $I = W_{max} - W$ ...(6.8)

This is also sometimes referred to as '***degradation***' or '***dissipation***'.

For a *non-flow process* between the equilibrium states, when the system exchanges heat only with environment, irreversibility (*per unit mass*),

$$i = [(u_1 - u_2) - T_0(s_1 - s_2)] - [(u_1 - u_2) + Q]$$
$$= T_0 (s_2 - s_1) - Q$$
$$= T_0 (\Delta s)_{system} + T_0 (\Delta s)_{surr.}$$

i.e., $$i = T_0 [(\Delta s)_{system} + (\Delta s)_{surr.}] \quad ...(6.9)$$

$\therefore$ $$i \geq 0$$

Similarly, for *steady flow-process*

$$i = W_{max} - W \text{ (per unit mass)}$$
$$= \left[\left(b_1 + \frac{C_1^2}{2} + gZ_1\right) - \left(b_2 + \frac{C_2^2}{2} + gZ_2\right)\right]$$
$$- \left[\left(h_1 + \frac{C_1^2}{2} + gZ_1\right) - \left(h_2 + \frac{C_2^2}{2} + gZ_2\right) + Q\right]$$

$$= T_0\,(s_2 - s_1) - Q$$
$$= T_0\,(\Delta s)_{system} + T_0\,(\Delta s)_{surr.}$$

i.e., $$i = T_0\,(\Delta s_{system} + \Delta s_{surr.})$$

The same expression for irreversibility applies to both flow and non-flow processes.

The quantity $T_0\,(\Delta s_{system} + \Delta s_{surr.})$ represents (per unit mass) an increase in unavailable energy (or energy).

6.8. EFFECTIVENESS

Effectiveness *is defined as the ratio of actual useful work to the maximum useful work.* The useful output of a system is given by the increase of availability of the surroundings.

$$\text{Effectiveness,} \quad \epsilon = \frac{\text{Increase of availability of surroundings}}{\text{Loss of availability of the system}} \qquad ...(6.10)$$

For a compression or heating process the effectiveness is given by

$$\epsilon = \frac{\text{Increase of availability of the system}}{\text{Loss of availability of the surroundings}}$$

or $$\epsilon = \frac{W_{useful}}{W_{max\,.\,useful}} \qquad ...[6.10\,(a)]$$

The *effectiveness of an actual process is always less than unity.* Thus effectiveness of a process is the measure of the extent to which advantage has been taken of an opportunity to obtain useful work.

Example 6.1. *One kg of air is compressed polytropically from 1 bar pressure and temperature of 300 K to a pressure of 6.8 bar and temperature of 370 K. Determine the irreversibility if the sink temperature is 293 K. Assume R = 0.287 kJ/kg K, c_p = 1.004 kJ/kg K and c_v = 0.716 kJ/kg K.* **(U.P.S.C.)**

Solution. Irreversibility $I = W_{max} - W_{act}$

$$-W_{max} = \text{Change in internal energy} - T_0 \times \text{Change in entropy}$$

or $$-W_{max} = (u_2 - u_1) - T_0(s_2 - s_1) = W_{rev}$$

or $$-W_{max} = c_v(T_2 - T_1) - T_0[c_p \ln (T_2/T_1) - R \ln (p_2/p_1)]$$
$$= 0.716(370 - 300) - 293 \times [1.005 \ln (370/300) - 0.287 \ln (6.8/1)]$$

or $$W_{max} = -149.53 \text{ kJ/kg} = W_{rev}$$

(negative sign indicates that work is done on air)

The index of compression 'n' is given by

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{[(n-1)/n]}$$

or $$\frac{n-1}{n} = \frac{\ln (T_2/T_1)}{\ln (p_2/p_1)} = \frac{\ln (370/300)}{\ln (6.8/1)}$$

or $$n = 1.123$$

$$W_{actual} = \frac{mR(T_1 - T_2)}{n-1} = \frac{1 \times 0.287(300 - 370)}{1.123 - 1} = -163.33 \text{ kJ/kg}$$

$$I = W_{rev} - W_{act} = -149.53 - (-163.33) = \mathbf{13.8\ kJ/kg.} \quad \textbf{(Ans.)}$$

Example 6.2. *A system at 500 K receives 7200 kJ/min from a source at 1000 K. The temperature of atmosphere is 300 K. Assuming that the temperatures of system and source remain constant during heat transfer find out :*

(*i*) *The entropy produced during heat transfer ;*
(*ii*) *The decrease in available energy after heat transfer*.

Solution. Refer Fig. 6.7.

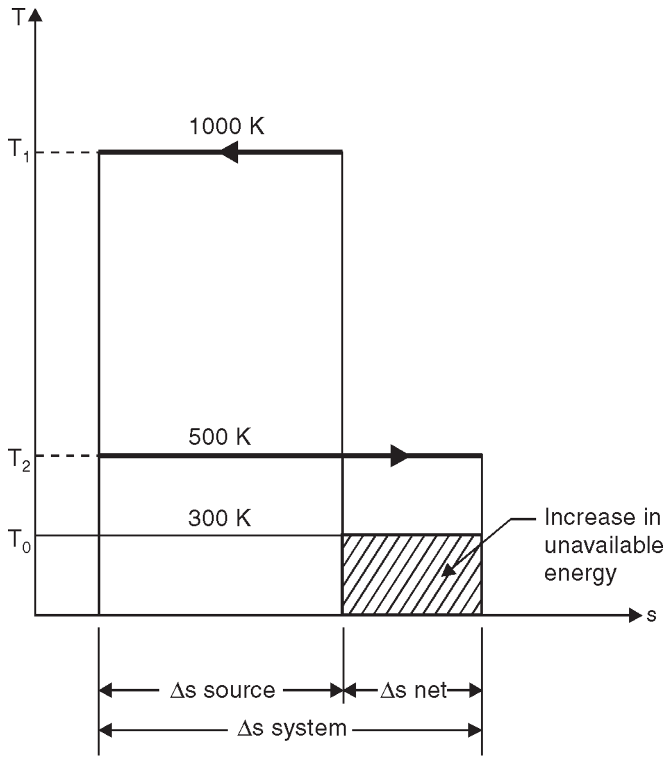


Fig. 6.7

Temperature of source, T_1 = 1000 K
Temperature of system, T_2 = 500 K
Temperature of atmosphere, T_0 = 300 K
Heat received by the system, Q = 7200 kJ/min.

(*i*) **Net change of entropy :**

Change in entropy of the *source* during heat transfer

$$= \frac{-Q}{T_1} = \frac{-7200}{1000} = -7.2 \text{ kJ/min-K}$$

Change in entropy of the *system* during heat transfer

$$= \frac{Q}{T_2} = \frac{7200}{500} = 14.4 \text{ kJ/min-K}$$

The net change of entropy, ΔS = – 7.2 + 14.4 = **7.2 kJ/min-K. (Ans.)**

(*ii*) **Decrease in available energy :**

Available energy with source

$$= (1000 - 300) \times 7.2 = 5040 \text{ kJ}$$

Available energy with the system

$$= (500 - 300) \times 14.4 = 2880 \text{ kJ}$$

$\therefore$ **Decrease in available energy** = 5040 – 2880 = **2160 kJ. (Ans.)**

$$\left[\begin{array}{l}\text{Also, increase in available energy} \\ \quad = T_0(S_2 - S_1) = T_0\,\Delta S \\ \quad = 300 \times 7.2 = 2160 \text{ kJ}\end{array}\right]$$

Example 6.3. *8 kg of air at 650 K and 5.5 bar pressure is enclosed in a closed system. If the atmosphere temperature and pressure are 300 K and 1 bar respectively, determine :*

(*i*) *The availability if the system goes through the ideal work producing process.*

(*ii*) *The availability and effectiveness if the air is cooled at constant pressure to atmospheric temperature without bringing it to complete dead state. Take* c_v *= 0.718 kJ/kg K ;* c_p*= 1.005 kJ/kg K.*

Solution. Mass of air, m = 8 kg

Temperature, T_1 = 650 K

Pressure, p_1 = 5.5 bar

Atmospheric pressure, p_0 = 1 bar

Atmospheric temperature, T_0 = 300 K

For air : c_v = 0.718 kJ/kg K ; c_p = 1.005 kJ/kg K.

(*i*) Change in available energy (for bringing the system to dead state),

$$= m[(u_1 - u_0) - T_0 \Delta s]$$

Also
$$\Delta s = c_v \log_e \left(\frac{T_1}{T_0}\right) + R \log_e \frac{v_1}{v_0}$$

Using the ideal gas equation,

$$\frac{p_1 v_1}{T_1} = \frac{p_0 v_0}{T_0}$$

$$\therefore \quad \frac{v_0}{v_1} = \frac{p_1}{p_0} \cdot \frac{T_0}{T_1} = \frac{5.5}{1} \times \frac{300}{650} = 2.54$$

$$\therefore \quad \Delta s = 0.718 \log_e \left(\frac{650}{300}\right) + 0.287 \log_e \left(\frac{1}{2.54}\right)$$

$$= 0.555 + (-0.267) = 0.288 \text{ kJ/kg K}$$

$\therefore$ Change in available energy

$$= m[(u_1 - u_0) - T_0 \Delta s] = m[c_v(T_1 - T_0) - T_0 \Delta s]$$

$$= 8[0.718(650 - 300) - 300 \times 0.288] = 1319.2 \text{ kJ}$$

Loss of availability per unit mass during the process

$$= p_0\,(v_0 - v_1) \text{ per unit mass}$$

Total loss of availability = $p_0(V_0 - V_1)$

But
$$V_1 = \frac{mRT_1}{p_1} = \frac{8 \times 287 \times 650}{5.5 \times 10^5} = 2.713 \text{ m}^3 \qquad \left[\because\ pV = mRT \text{ or } V = \frac{mRT}{p}\right]$$

and
$$V_0 = 2.54 \times 2.713 = 6.891 \text{ m}^3$$

$\therefore$ **Loss of availability** $= \dfrac{1 \times 10^5}{10^3}$ (6.891 – 2.713) = **417.8 kJ. (Ans.)**

(*ii*) Heat transferred during cooling (constant pressure) process

$= m.\ c_p\ (T_1 - T_0)$

$= 8 \times 1.005\ (650 - 300) = 2814$ kJ

Change in entropy during cooling

$$\Delta s = mc_p \log_e \left(\frac{T_1}{T_0}\right)$$

$$= 8 \times 1.005 \times \log_e \left(\frac{650}{300}\right) = 6.216 \text{ kJ/K}$$

Unavailable energy $= T_0\ \Delta S$

$= 300 \times 6.216 = 1864.8$ kJ

Available energy $= 2814 - 1864.8 =$ **949.2 kJ. (Ans.)**

Effectiveness, $\in = \dfrac{\text{Available energy}}{\text{Change in available energy}}$

$$= \frac{949.2}{1319.2} = \mathbf{0.719.}\quad \textbf{(Ans.)}$$

☞**Example 6.4.** *In a power station, the saturated steam is generated at 200°C by transferring the heat from hot gases in a steam boiler. Find the increase in total entropy of the combined system of gas and water and increase in unavailable energy due to irreversible heat transfer. The gases are cooled from 1000°C to 500°C and all the heat from gases goes to water. Assume water enters the boiler at saturated condition and leaves as saturated steam.*

Take : c_{pg} *(for gas) = 1.0 kJ/kg K,* h_{fg} *(latent heat of steam at 200°C) = 1940.7 kJ/kg.*

Atmospheric temperature = 20°C.

Obtain the results on the basis of 1 kg of water.

Solution. Refer Fig. 6.8.

Temperature of saturation steam $= 200 + 273 = 473$ K

Initial temperature of gases $= 1000 + 273 = 1273$ K

Final temperature of gases $= 500 + 273 = 773$ K

For gases : $c_{pg} = 1$ kJ/kg K

Latent heat of steam of 200°C

saturation temperature, $h_{fg} = 1940.7$ kJ/kg

Atmospheric temperature $= 20 + 273 = 293$ K

Heat lost by gases = Heat gained by 1 kg saturated water when it is converted to steam at 200°C.

$\therefore$ $m_g c_{pg}\ (1273 - 773) = 1940.7$

[where m_g = mass of gases, c_{pg} = specific heat of gas at constant pressure]

i.e., $$m_g = \frac{1940.7}{1.0 \times (1273 - 773)} = 3.88 \text{ kg}$$

Change of entropy of m_g kg of gas,

$$(\Delta S)_g = m_g\ c_{pg} \log_e \left(\frac{773}{1273}\right)$$

$$= 3.88 \times 1.0 \times \log_e \left(\frac{773}{1273}\right) = -\ 1.935 \text{ kJ/K}$$

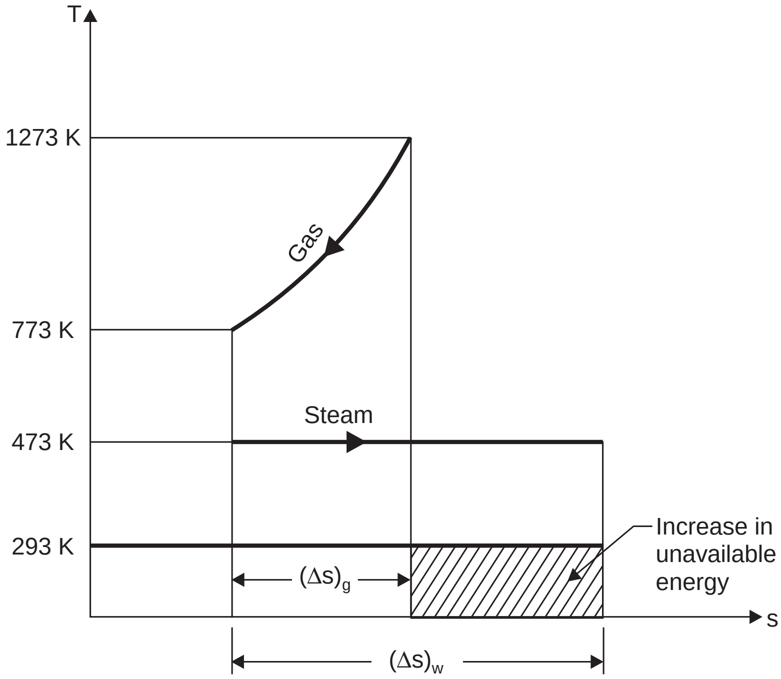


Fig. 6.8

Change of entropy of water (per kg) when it is converted into steam,

$$(\Delta s)_w = \frac{h_{fg}}{T_s} = \frac{1940.7}{(200+273)} = 4.103 \text{ kJ/kg K.}$$

Net change in entropy due to heat transfer

$$= -\,1.935 + 4.103 = \mathbf{2.168 \text{ kJ/K.}} \quad \textbf{(Ans.)}$$

Increase in unavailable energy due to heat transfer

= 293 × 2.168, *i.e.*, cross hatched area

= 635.22 kJ per kg of steam formed. (Ans.)

Example 6.5. *3 kg of gas (c_v = 0.81 kJ/kg K) initially at 2.5 bar and 400 K receives 600 kJ of heat from an infinite source at 1200 K. If the surrounding temperature is 290 K, find the loss in available energy due to above heat transfer.*

Solution. Refer Fig. 6.9.

Mass of gas, m_g = 3 kg

Initial pressure of gas = 2.5 bar

Initial temperature, T_1' = 400 K

Quantity of heat received by gas, Q = 600 kJ

Specific heat of gas, c_v = 0.81 kJ/kg K

Surrounding temperature = 290 K

Temperature of infinite source, T_1 = 1200 K

Heat received by the gas is given by,

$$Q = m_g c_v\,(T_2' - T_1')$$

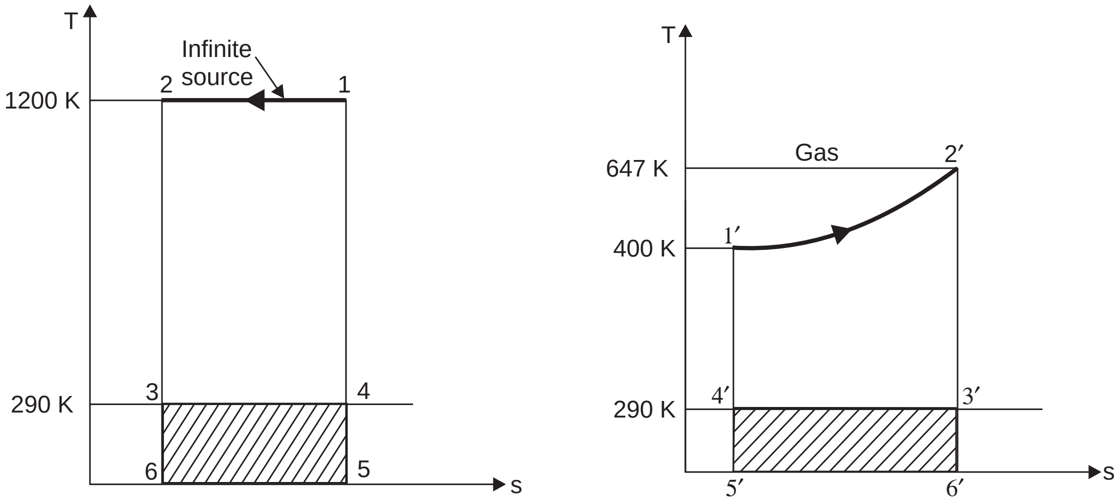


Fig. 6.9

$$600 = 3 \times 0.81\ (T_2' - 400)$$

$$\therefore \quad T_2' = \frac{600}{3 \times 0.81} + 400 = 646.9 \text{ K say } 647 \text{ K}$$

Available energy with the source

$$= \text{area } 1\text{-}2\text{-}3\text{-}4\text{-}1$$

$$= (1200 - 290) \times \frac{600}{1200} = 455 \text{ kJ}$$

Change in entropy of the gas

$$= m_g c_v \log_e \left(\frac{T_2'}{T_1'}\right) = 3 \times 0.81 \times \log_e \left(\frac{647}{400}\right) = 1.168 \text{ kJ/K}$$

Unavailability of the gas $= \text{area } 3'\text{-}4'\text{-}5'\text{-}6'\text{-}3'$

$$= 290 \times 1.168 = 338.72 \text{ kJ}$$

Available energy with the gas

$$= 600 - 338.72 = 261.28 \text{ kJ}$$

∴ **Loss in available energy due to heat transfer**

= 455 – 261.28 = **193.72 kJ. (Ans.)**

☞**Example 6.6.** *Calculate the unavailable energy in 60 kg of water at 60°C with respect to the surroundings at 6°C, the pressure of water being 1 atmosphere.*

Solution. Refer Fig. 6.10.

Mass of water, $m = 60$ kg

Temperature of water, $T_1 = 60 + 273 = 333$ K

Temperature of surroundings, $T_0 = 6 + 273 = 279$ K

Pressure of water, $p = 1$ atm.

If the water is cooled at a constant pressure of 1 atm. from 60°C to 6°C the heat given up may be used as a *source for a series of Carnot engines each using the surroundings as a sink.* It is assumed that the amount of energy received by any engine is small relative to that in the source and temperature of the source does not change while heat is being exchanged with the engine.

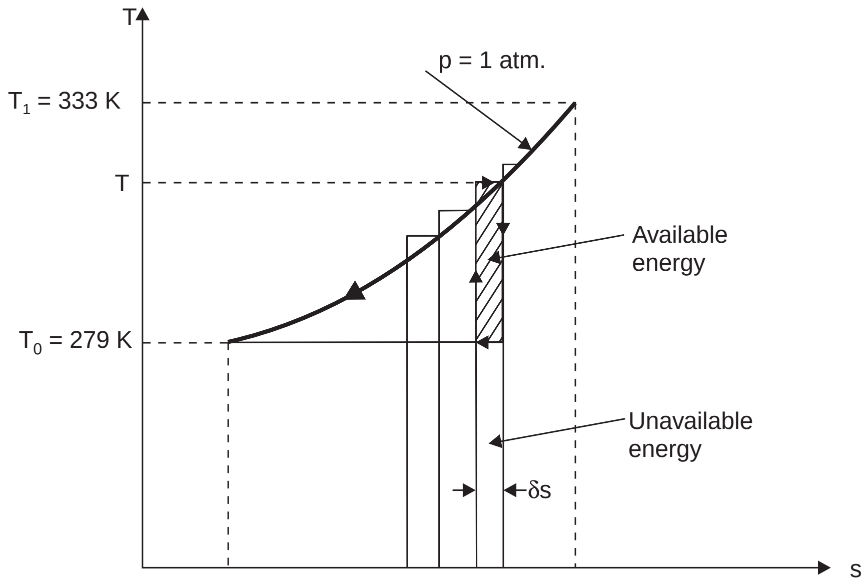


Fig. 6.10

Consider that the source has fallen to temperature T, at which level there operates a Carnot engine which takes in heat at this temperature and rejects heat at T_0 = 279 K. If δs is the entropy change of water, the work obtained is

$$\delta W = -\, m(T - T_0)\,\delta s$$

where δs is negative.

$$\therefore \qquad \delta W = -\,60\,(T - T_0)\,\frac{c_p \delta T}{T} = -\,60\,c_p\left(1 - \frac{T_0}{T}\right)\delta T$$

With a very great number of engines in the series, the total work (maximum) obtainable when the water is cooled from 333 K to 279 K would be

$$W_{max} = \text{Available energy}$$

$$= -\,\lim.\sum_{333}^{279} 60\,c_p\left(1 - \frac{T_0}{T}\right)\delta T$$

$$= \int_{279}^{333} 60\,c_p\left(1 - \frac{T_0}{T}\right)dT$$

$$= 60\,c_p\left[(333 - 279) - 279\log_e\left(\frac{333}{279}\right)\right]$$

$$= 60 \times 4.187\,(54 - 49.36) = 1165.7 \text{ kJ}$$

Also, $Q_1 = 60 \times 4.187 \times (333 - 279) = 13565.9$ kJ

$\therefore$ **Unavailable energy** = $Q_1 - W_{max}$

= 13565.9 – 1165.7 = **12400.2 kJ. (Ans.)**

Example 6.7. *15 kg of water is heated in an insulated tank by a churning process from 300 K to 340 K. If the surrounding temperature is 300 K, find the loss in availability for the process.*

Solution. Mass of water, m = 15 kg

Temperature, T_1 = 340 K

Surrounding temperature, $T_0 = 300$ K

Specific heat of water, $c_p = 4.187$ kJ/kg K

Loss in availability :

Work added during churning

= Increase in enthalpy of the water

= 15 × 4.187 × (340 – 300) = 2512.2 kJ

Now the energy in the water = 2512.2 kJ

The availability out of this energy is given by

$$m[(u_1 - u_0) - T_0\,\Delta s]$$

where $$\Delta s = c_p \log_e \left(\frac{T_1}{T_0}\right)$$

$\therefore$ $$\Delta s = 4.187 \log_e \left(\frac{340}{300}\right) = 0.524 \text{ kJ/kg K}$$

$\therefore$ Available energy

$$= m\,[c_v\,(T_1 - T_0) - T_0\,\Delta s]$$

$$= 15\,[4.187\,(340 - 300) - 300 \times 0.524] = 158.7 \text{ kJ}$$

$\therefore$ **Loss in availability**

= 2508 – 158.7 = **2349.3 kJ. (Ans.)**

This shows that *conversion of work into heat is highly irreversible process* (since out of 2512.2 kJ of work energy supplied to increase the temperature, only 158.7 kJ will be available again for conversion into work).

Example 6.8. *5 kg of air at 550 K and 4 bar is enclosed in a closed system.*

(*i*) *Determine the availability of the system if the surrounding pressure and temperature are 1 bar and 290 K respectively.*

(*ii*) *If the air is cooled at constant pressure to the atmospheric temperature, determine the availability and effectiveness.*

Solution. Mass of air, $m = 5$ kg

Temperature, $T_1 = 550$ K

Pressure, $p_1 = 4 \text{ bar} = 4 \times 10^5 \text{ N/m}^2$

Temperature, $T_2 = T_0 = 290$ K

Pressure, $p_2 = p_0 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$.

(*i*) **Availability of the system :**

Availability of the system is

$$= m[(u_1 - u_0) - T_0(s_1 - s_0)] = m[c_v(T_1 - T_0) - T_0\,\Delta s]$$

$$\Delta s = c_p \log_e \frac{T_1}{T_0} - R \log_e \left(\frac{p_1}{p_0}\right)$$

$$= 1.005 \log_e \left(\frac{550}{290}\right) - 0.287 \log_e \left(\frac{4}{1}\right)$$

$$= 0.643 - 0.397 = 0.246 \text{ kJ/kg K}$$

$\therefore$ **Availability of the system**

$$= m\,[c_v\,(T_1 - T_0) - T_0 \Delta s]$$

= 5[0.718 (550 – 290) – 290 × 0.246] = **576.7 kJ. (Ans.)**

(*ii*) Heat transferred during cooling

$$Q = m \times c_p \times (T_1 - T_0)$$
$$= 5 \times 1.005 \times (550 - 290)$$
$$= 1306.5 \text{ kJ} \quad \text{...... heat lost by the system}$$

Change of entropy during cooling

$$\Delta S = m \times c_p \times \log_e \left(\frac{T_1}{T_0}\right)$$
$$= 5 \times 1.005 \times \log_e \left(\frac{550}{290}\right) = 3.216 \text{ kJ/K}$$

Unavailable portion of this energy

$$= T_0 \,.\, (\Delta S) = 290 \times 3.216 = 932.64 \text{ kJ}$$

$\therefore$ **Available energy** $= 1306.5 - 932.64 =$ **373.86 kJ. (Ans.)**

Effectiveness, $\epsilon = \dfrac{\text{Available energy}}{\text{Availability of the system}} = \dfrac{373.86}{576.7}$

= **0.648 or 64.8%. (Ans.)**

Example 6.9. *Air at the rate of 25 kg/min is compressed in a centrifugal air compressor from 1 bar to 2 bar. The temperature increases from 15°C to 100°C during compression. Determine actual and minimum power required to run the compressor. The surrounding air temperature is 15°C.*

Neglect the heat interaction between the compressor and surroundings and changes in potential and kinetic energy.

Take for air, c_p *= 1.005 kJ/kg K, R = 0.287 kJ/kg K.*

Solution. Rate of flow of air, m = 25 kg/min.

Initial pressure, p_1 = 1 bar

Final pressure, p_2 = 2.0 bar

Initial temperature, $T_1 = T_0 = 15 + 273 = 288$ K

Final temperature, $T_2 = 100 + 273 = 373$ K.

Applying energy equation to compressor,

$$W_{actual} = h_2 - h_1 \quad [\text{as } Q = 0,\ \Delta PE = 0,\ \Delta KE = 0]$$
$$= c_p\,(T_2 - T_1) = 1.005\,(373 - 288) = 85.4 \text{ kJ/kg}$$

Total actual work done/min

$$= 25 \times 85.4 = 2135 \text{ kJ/min}$$
$$= \frac{2135}{60} = 35.58 \text{ kJ/s} = 35.58 \text{ kW}$$

The *minimum work required is given by the increase in availability of the air stream.*

$$W_{min} = b_2 - b_1 = (h_2 - h_1) - T_0\,(s_2 - s_1)$$
$$s_2 - s_1 = c_p \log_e \left(\frac{T_2}{T_1}\right) - R \log_e \left(\frac{p_2}{p_1}\right) \quad \text{...per unit mass}$$
$$= 1.005 \log_e \left(\frac{373}{288}\right) - 0.287 \log_e \left(\frac{2.0}{1}\right)$$
$$= 0.2599 - 0.1989 = 0.061 \text{ kJ/kg K}$$

$\therefore$
$$W_{min} = (h_2 - h_1) - T_0(s_2 - s_1)$$

$= 85.4 - 288 \times 0.061 = 67.8$ kJ/kg

∴ **Minimum work required**

$$\frac{25 \times 67.8}{60} = 28.25 \text{ kJ/s} = \textbf{28.25 kW.} \quad \textbf{(Ans.)}$$

Example 6.10. *1 kg of oxygen at 1 bar and 450 K is mixed with 1 kg of hydrogen at the same temperature and pressure by removing the diaphragm as shown in Fig. 6.11. Determine the loss in availability if the surrounding temperature is 290 K.*

Assume that the system is fully isolated.

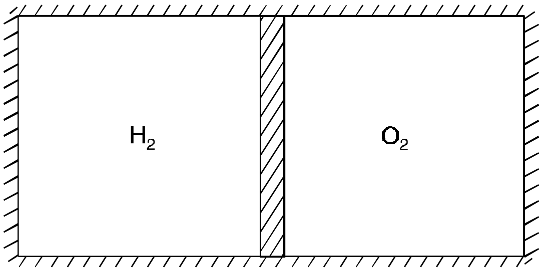


Fig. 6.11

Solution. Mass of oxygen, $m_{O_2} = 1$ kg

Mass of hydrogen, $m_{H_2} = 1$ kg

Pressure, $p = 1$ bar $= 1 \times 10^5$ N/m^2

Temperature, $T_{O_2} = T_{H_2} = 450$ K

Surrounding temperature $= 290$ K

Characteristic gas constant of O_2,

$$R_{O_2} = \frac{R_0}{M_{O_2}} = \frac{8314}{32} = 259.6 \text{ J/kg K}$$

Now to find volume of O_2 using the relation,

$$pv = mRT$$

$$v = \frac{mRT}{p} = \frac{1 \times 259.6 \times 450}{1 \times 10^5} = 1.168 \text{ m}^3$$

i.e., $v_{O_2} = 2.336$ m^3

Characteristic gas constant of H_2,

$$R_{H_2} = \frac{R_0}{M_{H_2}} = \frac{8314}{2} = 4157 \text{ J/kg K}$$

Volume of H_2, $v_{H_2} = \dfrac{1 \times 4157 \times 450}{1 \times 10^5} = 18.706$ m^3

Total volume after mixing

$$= v_{O_2} + v_{H_2} = 1.168 + 18.706$$

$$= 19.874 \text{ m}^3$$

The partial pressure of each gas changes after the mixing even though the temperature is the same due to increase in volume.

Change in entropy of *oxygen*

$$= R_{O_2} \log_e \frac{v_2}{v_1}$$

$$= 259.6 \log_e \left(\frac{19.874}{1.168}\right) = 735.7 \text{ J/K}$$

Change in entropy of hydrogen

$$= R_{H_2} \log_e \frac{v_2}{v_1} = 4157 \log_e \left(\frac{19.874}{18.706}\right)$$

$$= 251.78 \text{ J/K}$$

Net change in entropy,

$$\Delta S = 735.7 + 251.78 = 987.48 \text{ J/K}$$

Loss in availability

$$= T_0 \Delta S = 290 \times 987.48 \text{ J} = \frac{290 \times 987.48}{10^3} \text{ kJ} = 286.36 \text{ kJ}$$

i.e., **Loss in availability = 286.36 kJ. (Ans.)**

☞**Example 6.11.** *Calculate the decrease in available energy when 20 kg of water at 90°C mixes with 30 kg of water at 30°C, the pressure being taken as constant and the temperature of the surroundings being 10°C.*

Take c_p of water as 4.18 kJ/kg K.

Solution. Temperature of surrounding, $T_0 = 10 + 273 = 283$ K

Specific heat of water, $c_p = 4.18$ kJ/kg K

The available energy of a system of mass m, specific heat c_p, and at temperature T, is given by,

Available energy, $\text{A.E.} = mc_p \int_{T_0}^{T} \left(1 - \frac{T_0}{T}\right) dT$

Now, available energy of 20 kg of water at 90°C,

$$(\text{A.E.})_{20 \text{ kg}} = 20 \times 4.18 \int_{(10+273)}^{(90+273)} \left(1 - \frac{283}{T}\right) dT$$

$$= 83.6 \left[(363 - 283) - 283 \log_e \left(\frac{363}{283}\right)\right]$$

$$= 83.6 (80 - 70.45) = 798.38 \text{ kJ}$$

Available energy of 30 kg of water at 30°C,

$$(\text{A.E.})_{30 \text{ kg}} = 30 \times 4.18 \int_{(10+273)}^{(30+273)} \left(1 - \frac{283}{T}\right) dT$$

$$= 30 \times 4.18 \left[(303 - 283) - 283 \log_e \left(\frac{303}{283}\right)\right]$$

$$= 125.4 (20 - 19.32) = 85.27 \text{ kJ}$$

Total available energy,

$$(\text{A.E.})_{\text{total}} = (\text{A.E.})_{20 \text{ kg}} + (\text{A.E.})_{30 \text{ kg}}$$

$$= 798.38 + 85.27 = 883.65 \text{ kJ}$$

If t°C is the final temperature after mixing, then

$$20 \times 4.18 \times (90 - t) = 30 \times 4.18 (t - 30)$$

or $20(90 - t) = 30\,(t - 30)$

$$\therefore \quad t = \frac{20 \times 90 + 30 \times 30}{20 + 30} = 54°\text{C}$$

Total mass after mixing = 20 + 30 = 50 kg

Available energy of 50 kg of water at 54°C

$$(\text{A.E.})_{50\ \text{kg}} = 50 \times 4.18 \left[(327 - 283) - 283 \log_e \left(\frac{327}{283}\right)\right]$$

$$= 209\,(44 - 40.89) = 649.99 \text{ kJ}$$

∴ **Decrease in available energy due to mixing**

= Total energy before mixing – Total energy after mixing

= 883.65 – 649.99 = **233.66 kJ. (Ans.)**

☞ **Example 6.12.** *In an heat exchanger (parallel flow type) waters enter at 50°C and leaves at 70°C while oil (specific gravity = 0.82, specific heat = 2.6 kJ/kg K) enters at 240°C and leaves at 90°C. If the surrounding temperature is 27°C determine the loss in availability on the basis of one kg of oil per second.*

Solution. Refer Fig. 6.12.

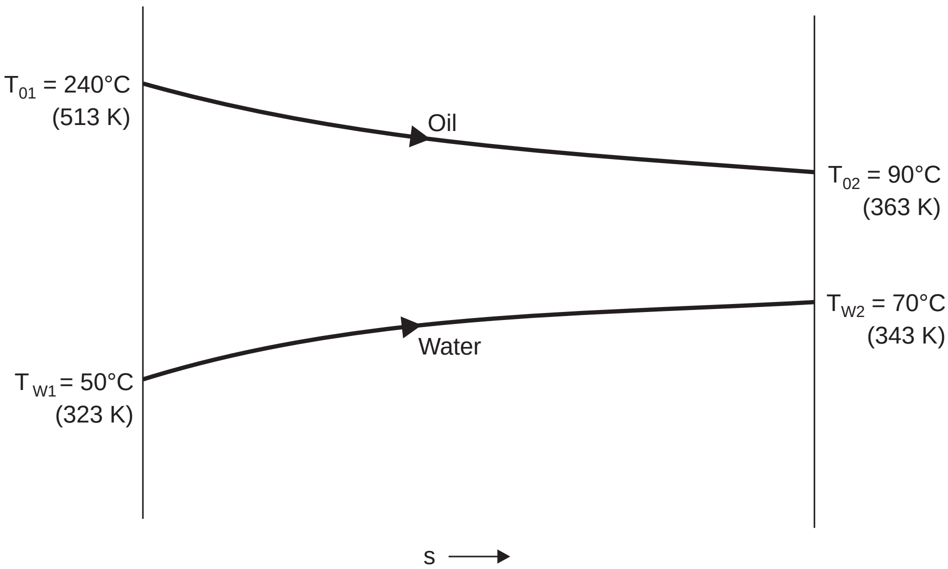


Fig. 6.12

Inlet temperature of water, T_{w_1} = 50°C = 323 K

Outlet temperature of water, T_{w_2} = 70°C = 343 K

Inlet temperature of oil, T_{o_1} = 240°C = 513 K

Outlet temperature of oil, T_{o_2} = 90°C = 363 K

Specific gravity of oil = 0.82

Specific heat of oil = 2.6 kJ/kg K

Surrounding temperature, T_0 = 27 + 273 = 300 K.

Loss in availability :

Consider one kg of oil.

Heat lost by oil = Heat gained by water

$$m_0 \times c_{po} \times (T_{o_1} - T_{o_2}) = m_w \times c_{pw} \times (T_{w_2} - T_{w_1})$$

where c_{po} = Specific heat of oil (2.6 kJ/kg K),

c_{pw} = Specific heat of water (4.18 kJ/kg K), and

m_o = Mass of oil (= 1 kg).

m_w = Mass of water (= ?)

$\therefore \quad 1 \times 2.6 \times (513 - 363) = m_w \times 4.18 \times (343 - 323)$

or $\quad 390 = 83.6\ m_w$ or $m_w = 4.66$ kg

Entropy change of water

$$= m_w\, c_{pw} \log_e \frac{T_{w_2}}{T_{w_1}} = 4.66 \times 4.18 \times \log_e \left(\frac{343}{323}\right) = 1.17 \text{ kJ/K}$$

Entropy change of oil

$$= m_o c_{po} \log_e \left(\frac{T_{o_2}}{T_{o_1}}\right) = 1 \times 2.6 \log_e \left(\frac{363}{513}\right) = -\ 0.899 \text{ kJ/K}$$

Change in availability of water

$$= m_w[c_{pw}(T_{w_2} - T_{w_1})] - T_o\ (\Delta S)_w$$

$$= 4.66[(4.18\ (343 - 323)] - 300 \times 1.17 = 38.57 \text{ kJ}$$

+ve sign indicates an *increase in availability*

Change in availability of oil

$$= m_o[c_{po}(T_{o_2} - T_{o_1})] - T_0\ (\Delta S)_0]$$

$$= 1[2.6(363 - 513)] - 300 \times (-\ 0.899) = -\ 120.3 \text{ kJ/K}$$

$\therefore$ **Loss in availability**

$= -\ 120.3 + 38.57 = -$ **81.73 kJ. (Ans.)**

(–ve sign indicates the **loss**).

Example 6.13. *1 kg of ice at 0°C is mixed with 12 kg of water at 27°C. Assuming the surrounding temperature as 15°C, calculate the net increase in entropy and unavailable energy when the system reaches common temperature :*

Given : Specific heat of water = 4.18 kJ/kg K ; specific heat of ice = 2.1 kJ/kg K and enthalpy of fusion of ice (latent heat) = 333.5 kJ/kg.

Solution. Mass of ice, m_{ice} = 1 kg

Temperature of ice, T_{ice} = 0 + 273 = 273 K

Mass of water, m_{water} = 12 kg

Temperature of water, T_{water} = 27 + 273 = 300 K

Surrounding temperature, T_0 = 15 + 273 = 288 K

Specific heat of water = 4.18 kJ/kg K

Specific heat of ice = 2.1 kJ/kg K

Latent heat of ice = 333.5 kJ/kg

Let T_c = *common temperature* when heat flows between ice and water stops.

Heat lost by water = Heat gained by ice

i.e., $12 \times 4.18(300 - T_c) = 4.18(T_c - 273) + 333.5$

or $15048 - 50.16T_c = 4.18T_c - 1141.14 + 333.5$

or $54.34\ T_c = 15855.64$

$\therefore$ $T_c = 291.8$ K or 18.8°C.

Change of entropy of water $= 12 \times 4.18 \log_e \left(\dfrac{291.8}{300}\right) = -1.39$ kJ/K

Change of entropy of ice $= 1 \times 4.18 \log_e \left(\dfrac{291.8}{273}\right) + \dfrac{333.5}{273} = 1.499$ kJ/K

Net change of entropy, $\Delta S = -1.39 + 1.499 = 0.109$ kJ/K

Hence, **net increase in entropy = 0.109 kJ/K. (Ans.)**

Increase in unavailable energy = $T_0 \Delta S$ = 288 × 0.109 = **31.39 kJ. (Ans.)**

☞**Example 6.14.** *A vapour, in a certain process, while condensing at 400°C, transfers heat to water at 200°C. The resulting steam is used in a power cycle which rejects heat at 30°C.*

What is the fraction of the available energy in the heat transferred from the process vapour at 400°C that is lost due to the irreversible heat transfer at 200°C ?

Solution. Refer Fig. 6.13.

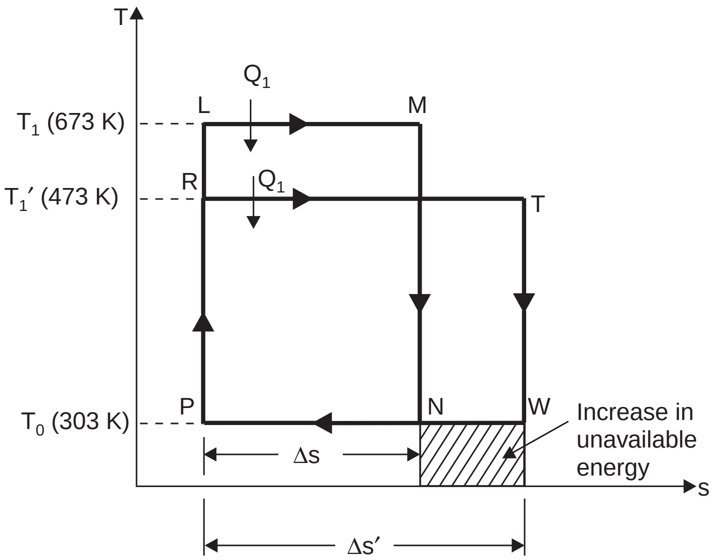


Fig. 6.13

Temperature of vapour, $T_1 = 400 + 273 = 673$ K

Temperature of water, $T_2 = 200 + 273 = 473$ K

Temperature at which heat is rejected, $T_0 = 30 + 273 = 303$ K.

LMNP (Fig. 6.13) would have been the power cycle, if there was no temperature difference between the vapour condensing and the vapour evaporating, and the area under *NP* would have been the unavailable energy. *RTWP* is the power cycle when the vapour condenses at 400°C and the water evaporates at 200°C. The unavailable energy becomes the area under *PW*. Therefore, the increase in unavailable energy due to irreversible heat transfer is represented by the area under *NW*.

Now,
$$Q_1 = T_1\Delta s = T_1'\Delta s'$$

$$\frac{\Delta s'}{\Delta s} = \frac{T_1}{T_1'}$$

$$W = \text{Work done in cycle } LMNP$$
$$= (T_1 - T_0)\,\Delta s \quad \text{...per unit mass}$$
$$W' = \text{Work done in cycle } RTWP$$
$$= (T_1 - T_0)\,\Delta s' \quad \text{...per unit mass}$$

The fraction of energy that becomes unavailable due to irreversible heat transfer,

$$\frac{W - W'}{W} = \frac{(T_1 - T_0)\Delta s - (T_1 - T_0)\Delta s'}{(T_1 - T_0)\Delta s} = \frac{T_0\,(\Delta s' - \Delta s)}{(T_1 - T_0)\Delta s}$$

$$= \frac{T_0\left(\frac{\Delta s'}{\Delta s} - 1\right)}{(T_1 - T_0)} = \frac{T_0\left(\frac{T_1}{T_1'} - 1\right)}{(T_1 - T_0)}$$

$$= \frac{T_0\,(T_1 - T_1')}{T_1'\,(T_1 - T_0)} = \frac{303(673 - 473)}{473(673 - 303)} = 0.346$$

Hence **the fraction of energy that becomes unavailable = 0.346 or 34.6%. (Ans.)**

Example 6.15. *A liquid is heated at approximately constant pressure from 20°C to 80°C by passing it through tubes which are immersed in a furnace. The furnace temperature is constant at 1500°C. Calculate the effectiveness of the heating process when the atmospheric temperature is 15°C.*

Take specific heat of liquid as 6.3 kJ/kg K.

Solution. Initial temperature of fluid, $T_1 = 20 + 273 = 293$ K

Final temperature of fluid, $T_2 = 80 + 273 = 353$ K

Temperature of the furnace, $T_f = 1500 + 273 = 1773$ K

Atmospheric temperature, $T_0 = 15 + 273 = 288$ K

Specific heat of liquid, $c_{pl} = 6.35$ kJ/kg K

Increase of availability of the liquid

$$= b_2 - b_1 = (h_2 - h_1) - T_0(s_2 - s_1)$$

i.e.,
$$b_2 - b_1 = c_{pl}\,(T_2 - T_1) - T_0 \times c_{pl}\,\log_e \frac{T_2}{T_1}$$

$$= 6.3\,(353 - 293) - 288 \times 6.3 \times \log_e \left(\frac{353}{293}\right) = 39.98 \text{ kJ/kg}$$

Now, the heat rejected by the furnace = Heat supplied to the liquid, $(h_2 - h_1)$.

If this quantity of heat were supplied to a heat engine operating on the Carnot cycle its thermal efficiency would be,

$$\eta_{th} = \left(1 - \frac{T_0}{T_f}\right) = \left(1 - \frac{288}{1773}\right) = 0.837 \text{ (or 83.7\%)}$$

$\therefore$ Work which could be obtained from a heat engine

$$= \text{Heat supplied} \times \text{Thermal efficiency}$$

i.e., Possible work of heat engine = $(h_2 - h_1) \times 0.837$

The possible work from a heat engine is a measure of the loss of availability of the furnace.

∴ Loss of availability of surroundings

$$= (h_2 - h_1) \times 0.837 = c_{pl}\,(T_2 - T_1) \times 0.837$$
$$= 6.3\,(353 - 293) \times 0.837 = 316.38 \text{ kJ/kg}$$

Then, *effectiveness of the heating process*,

$$\in = \frac{\text{Increase of availability of the liquid}}{\text{Loss of availability of surroundings}}$$

$$= \frac{39.98}{316.38} = \mathbf{0.1263} \text{ or } \mathbf{12.63\%}. \quad \textbf{(Ans.)}$$

Note. *The very low value of effectiveness reflects the irreversibility of the transfer of heat through a large temperature difference.* If the furnace temperature were much lower then process would be much more effective, although the heat transferred to the liquid would remain the same.

Example 6.16. *Air at 20°C is to be heated to 50°C by mixing it in steady flow with a quantity of air at 100°C. Assuming that the mixing process is adiabatic and neglecting changes in kinetic and potential energy, calculate :*

(*i*) *The ratio of mass flow of air initially at 100°C to that initially at 20°C.*

(*ii*) *The effectiveness of heating process, if the atmospheric temperature is 20°C.*

Solution. (*i*) Let, x = ratio of mass flows.

Stream 1 = air at 20°C (T_1 = 20 + 273 = 293 K)

Stream 2 = air at 100°C (T_2 = 100 + 273 = 373 K)

Stream 3 = air at 50°C (T_3 = 50 + 273 = 323 K)

If, c_p = Specific heat of air constant pressure

Then $c_pT_1 + xc_pT_2 = (1 + x)c_pT_3$

or $c_pT_1 + xc_pT_2 = c_pT_3 + xc_pT_3$

or $xc_p(T_2 - T_3) = c_p(T_3 - T_1)$

i.e., $xc_p(373 - 323) = c_p(323 - 293)$

∴ $x = \dfrac{30}{50}$ = **0.6. (Ans.)**

(*ii*) Let the system considered be a stream of air of unit mass, heated from 20°C to 50°C.

Increase of availability of system

$$= b_3 - b_1 = (h_3 - h_1) - T_0(s_3 - s_1) = c_p(T_3 - T_1) - T_0(s_3 - s_1)$$
$$= 1.005(323 - 293) - 293(s_3 - s_1) \qquad [\because \; T_0 = 20 + 273 = 293 \text{ K}]$$

Also, $s_3 - s_1 = c_p \log_e \dfrac{T_3}{T_1} = 1.005 \log_e \dfrac{323}{293} = 0.0979$ kJ/kg K

∴ *Increase of availability of system*

$$= 1.005 \times 30 - 293 \times 0.0979 = 1.465 \text{ kJ/kg.}$$

The system, which is the air being heated, is 'surrounded' by the air stream being cooled. Therefore, the loss of availability of the surroundings is given by, $x(b_2 - b_3)$.

i.e., *Loss of availability of surroundings*

$$= x[(h_2 - h_3) - T_0(s_2 - s_3)] = 0.6[c_p(T_2 - T_3) - T_0(s_2 - s_3)]$$

$$= 0.6\left[1.005\,(373 - 323) - 293 \times 1.005 \log_e \left(\frac{373}{323}\right)\right]$$

$$= 0.6[50.25 - 42.38] = 4.722 \text{ kJ/kg}$$

$$\therefore \qquad \textit{Effectiveness} = \frac{\text{Increase of availability of system}}{\text{Loss of availability of surroundings}}$$

$$= \frac{1.465}{4.722} = \mathbf{0.31} \quad \text{or} \quad \mathbf{31\%.} \quad \textbf{(Ans.)}$$

The *low* figure for the effectiveness is an indication of the *highly irreversible nature of the mixing process.*

☞**Example 6.17.** *2.5 kg of air at 6 bar, 90°C expands adiabatically is a closed system until its volume is doubled and its temperature becomes equal to that of the surroundings which is at 1 bar, 5°C. For this process determine :*

(*i*) *The maximum work ;*

(*ii*) *The change in availability ;*

(*iii*) *The irreversibility.*

For air take : c_v *= 0.718 kJ/kg K, R = 0.287 kJ/kg K.*

Solution. Mass of air, $m = 2.5$ kg

Initial pressure of air, $p_1 = 6 \text{ bar} = 6 \times 10^5 \text{ N/m}^2$

Ratio of final to initial volume, $\frac{V_2}{V_1} = 2$

Initial temperature of air, $T_1 = 90 + 273 = 363$ K

Final pressure of air, $p_2 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$

Final temperature of air, $T_2 = T_0 = 5 + 273 = 278$ K

From the property relation

$$TdS = dU + pdV$$

or
$$dS = \frac{dU}{T} + \frac{pdV}{T}$$

$$dS = \frac{mc_v dT}{T} + \frac{mRdV}{V} \qquad \left[\because\ u = c_v\, dT \text{ and } pV = mRT \text{ or } \frac{p}{T} = \frac{mR}{V}\right]$$

$\therefore$ The entropy change of air between the initial and final states is

$$\int_1^2 dS = \int_1^2 \frac{mc_v dT}{T} + \int_1^2 \frac{mRdV}{V}$$

or
$$S_2 - S_1 = mc_v \log_e \frac{T_2}{T_1} + mR \log_e \frac{V_2}{V_1}$$

(*i*) **The maximum work, W_{max} :**

Also,
$$W_{max} = (U_1 - U_2) - T_0(S_1 - S_2)$$

$$= m\left[c_v(T_1 - T_2) + T_0\left(c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}\right)\right]$$

$$= 2.5\left[0.718(363 - 278) + 278\left(0.718 \log_e\left(\frac{278}{363}\right) + 0.287 \log_e 2\right)\right]$$

$$= 2.5[61.03 + 278(-0.1915 + 0.1989)] = 157.7 \text{ kJ}$$

Hence, **maximum work = 157.7 kJ. (Ans.)**

(*ii*) **The change in availability :**

The change in availability is given by,

$$A_1 - A_2 = (U_1 - U_2) - T_0(S_1 - S_2) + p_0(V_1 - V_2)$$
$$= W_{max} + p_0 (V_1 - V_2)$$
$$= 157.7 + p_0 (V_1 - 2V_1) = 157.7 - P_0V_1 \qquad \left[\begin{array}{l} p_1V_1 = mRT_1 \\ \because V_1 = \dfrac{mRT_1}{p_1} \end{array}\right]$$
$$= 157.7 - \frac{1 \times 10^5}{10^3}\left[\frac{2.5 \times (0.287 \times 1000) \times 363}{6 \times 10^5}\right] = 114.29 \text{ kJ}$$

Hence **change in availability = 114.29 kJ. (Ans.)**

(*iii*) **The irreversibility ; I :**

The irreversibility is given by

$$I = W_{max.\ useful} - W_{actual}$$

From the first law of thermodynamics,

$$W_{actual} = Q - \Delta U = - \Delta U = U_1 - U_2 \qquad [\because \ Q = 0 \ ... \text{ adiabatic process}]$$
$$I = (U_1 - U_2) - T_0(S_1 - S_2) - (U_1 - U_2)$$
$$= T_0(S_2 - S_1)$$
$$= T_0(\Delta S)_{system}$$

For adiabatic process, $(\Delta S)_{surr.} = 0$

$$I = T_0 \left[mc_v \log_e \frac{T_2}{T_1} + mR \log_e \frac{v_2}{v_1} \right]$$
$$= 278 \times 2.5 \left(0.718 \log_e \left(\frac{278}{363}\right) + 0.287 \log_e 2 \right)$$
$$= 695(- 0.1915 + 0.1989) = 5.143 \text{ kJ}$$

Hence, **the irreversibility = 5.143 kJ. (Ans.)**

☞**Example 6.18.** *In a turbine the air expands from 7 bar, 600°C to 1 bar, 250°C. During expansion 9 kJ/kg of heat is lost to the surroundings which is at 1 bar, 15°C. Neglecting kinetic energy and potential energy changes, determine per kg of air :*

(*i*) *The decrease in availability ;*

(*ii*) *The maximum work ;*

(*iii*) *The irreversibility.*

For air, take : $c_p = 1.005$ *kJ/kg K,* $h = c_pT$*, where* c_p *is constant.*

Solution. Mass of air considered = 1 kg

Pressure,	$p_1 = 7 \text{ bar} = 7 \times 10^5 \text{ N/m}^2$
Temperature,	$T_1 = 600 + 273 = 873 \text{ K}$
Pressure,	$p_2 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$
Temperature,	$T_2 = 250 + 273 = 523 \text{ K}$
Surrounding temperature,	$T_0 = 15 + 273 = 288 \text{ K}$

Heat lost to the surroundings during expansion,

$$Q = 9 \text{ kJ/kg.}$$

(*i*) From the property relation,

$$TdS = dH - Vdp$$

$$dS = \frac{dH}{T} - \frac{Vdp}{T}$$

$$= \frac{m.c_p dT}{T} - \frac{mRdp}{p}$$

or $$\int_1^2 dS = \int_1^2 \frac{m.c_p dT}{T} - \int_1^2 \frac{mRdp}{p}$$

or $$S_2 - S_1 = mc_p \log_e \frac{T_2}{T_1} - mR \log_e \frac{p_2}{p_1}$$

$$\left\{ \begin{array}{l} \because\ h = c_p T,\ dh = c_p dT \\ dH = mc_p dT \text{ and } pV = mRT \\ \text{or } V = \dfrac{mRT}{p} \end{array} \right\}$$

For 1 kg of air

$$s_2 - s_1 = c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}$$

Now, the change in availability is given by

$$b_1 - b_2 = (h_1 - T_0 s_1) - (h_2 - T_0 s_2)$$

$$= (h_1 - h_2) - T_0(s_1 - s_2)$$

$$= c_p (T_1 - T_2) - T_0 \left(R \log_e \frac{p_2}{p_1} - c_p \log_e \frac{T_2}{T_1} \right)$$

$$= 1.005(873 - 523) - 288 \left[0.287 \log_e \left(\frac{1}{7} \right) - 1.005 \log_e \left(\frac{523}{873} \right) \right]$$

$$= 351.75 - 288(-0.5585 + 0.5149) = 364.3 \text{ kJ/kg}$$

i.e., **Decrease in availability = 364.3 kJ/kg. (Ans.)**

(*ii*) **The maximum work,**

W_{max} = Change in availability = **364.3 kJ/kg. (Ans.)**

(*iii*) From steady flow energy equation,

$$Q + h_1 = W + h_2$$

$$W = (h_1 - h_2) + Q$$

$$= c_p (T_1 - T_2) + Q$$

$$= 1.005(873 - 523) + (-9) = 342.75 \text{ kJ/kg}$$

The irreversibility,

$$I = W_{max} - W$$

$= 364.3 - 342.75 =$ **21.55 kJ/kg. (Ans.)**

[Alternatively, $I = T_0 (\Delta S_{system} + \Delta S_{surr.})$

$$= 288 \left[1.005 + \log_e \left(\frac{523}{873} \right) - 0.287 \log_e \left(\frac{1}{7} \right) + \frac{9}{288} \right]$$

$$= 288 [-0.5149 + 0.5585 + 0.03125]$$

$$= 21.55 \text{ kJ/kg.}$$]

Example 6.19. *1 kg of air undergoes a polytropic compression from 1 bar and 290 K to 6 bar and 400 K. If the temperature and pressure of the surroundings are 290 K and 1 bar respectively, determine :*

(*i*) *The irreversibility ;*

(*ii*) *The effectiveness.*

Take for air : c_p *= 1.005 kJ/kg K,* c_v *= 0.718 kJ/kg K, R = 0.287 kJ/kg K.*

Solution. Mass of air, $m = 1$ kg

Initial temperature, $T_1 = T_0 = 290$ K

Final temperature, $T_2 = 400$ K

Initial pressure, $p_1 = p_0 = 1$ bar

Final pressure, $p_2 = 6$ bar.

(*i*) **The irreversibility, I :**

We know that reversible work,

$$W_{rev.} = \text{Change in internal energy} - T_0 \times \text{Change in entropy}$$

$$= (u_2 - u_1) - T_0 \left[c_p \log_e \left(\frac{T_2}{T_1}\right) - R \log_e \left(\frac{p_2}{p_1}\right)\right]$$

$$= c_v (T_2 - T_1) - T_0 \left[c_p \log_e \left(\frac{T_2}{T_1}\right) - R \log_e \left(\frac{p_2}{p_1}\right)\right]$$

$$= 0.718 (400 - 290) - 290 \left[1.005 \log_e \left(\frac{400}{290}\right) - 0.287 \log_e \left(\frac{6}{1}\right)\right]$$

$$= 78.98 - 290 (0.3232 - 0.5142)$$

$= 134.37$ kJ/kg (–) [–ve sign indicates that work is done on the air]

Actual work is given by

$$W_{actual} = \frac{mR(T_1 - T_2)}{n-1} = \frac{R(T_1 - T_2)}{n-1} \text{ as } m = 1 \text{ kg.}$$

The index n for the compression is given by :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\therefore \quad \frac{400}{290} = \left(\frac{6}{1}\right)^{\frac{n-1}{n}} \quad \text{or} \quad 1.379 = (6)^{\frac{n-1}{n}}$$

$$\log_e 1.379 = \left(\frac{n-1}{n}\right) \log_e 6 \quad \text{or} \quad 0.3213 = 1.7917 \left(\frac{n-1}{n}\right)$$

$$\therefore \quad \frac{n-1}{n} = \frac{0.3213}{1.7917} = 0.1793 \quad \text{or} \quad n = 1.218$$

$$\therefore \quad W_{actual} = \frac{0.287 (290 - 400)}{(1.218 - 1)} = -144.8 \text{ kJ}$$

(–ve sign means that the work is done on the air)

Now, **irreversibility** = $W_{rev} - W_{actual}$

= – 134.37 – (– 144.8) = **10.43 kJ. (Ans.)**

(*ii*) **The effectiveness, ∈ :**

$$\in = \frac{W_{rev.}}{W_{actual}} = \frac{-134.37}{-144.8}$$ **= 0.928** or **92.8%. (Ans.)**

Example 6.20. *A flywheel whose moment of inertia is 0.62 kg m^2 rotates at a speed 2500 r.p.m. in a large heat insulated system, the temperature of which is 20°C.*

(*i*) *If the K.E. of the flywheel is dissipated as frictional heat at the shaft bearings which have a water equivalent of 1.9 kg, find the rise in the temperature of the bearings when the flywheel has come to rest.*

(*ii*) *Calculate the greatest possible amount of the above heat which may be returned to the flywheel as high-grade energy, showing how much of the original K.E. is now unavailable. What would be the final r.p.m. of the flywheel, if it is set in motion with this available energy ?*

Solution. Moment of inertia of the flywheel, I = 0.62 kg m^2

Initial angular velocity of the flywheel, $\omega_1 = \dfrac{2\pi N_1}{60} = \dfrac{2\pi \times 2500}{60}$

$= 261.8$ rad/s.

Temperature of insulated system, $T_0 = 20 + 273 = 293$ K

Water equivalent of shaft bearings = 1.9 kg

(*i*) Initial available energy of the flywheel,

$$(\text{K.E.})_{initial} = \frac{1}{2} I\omega_1^2$$

$$= \frac{1}{2} \times 0.62 \times (261.8)^2 = 2.12 \times 10^4 \text{ N.m} = 21.2 \text{ kJ.}$$

When this K.E. is dissipated as frictional heat, if Δt is the temperature rise of the bearings, we have

Water equivalent of bearings × rise in temperature = 21.2

i.e., $(1.9 \times 4.18)\ \Delta t = 21.2$

or
$$\Delta t = \frac{21.2}{1.9 \times 4.18} = 2.67°\text{C}$$

Hence, **rise in temperature of bearings = 2.67°C. (Ans.)**

∴ **Final temperature of the bearings = 20 + 2.67 = 22.67°C.**

(*ii*) *The maximum amount of energy which may be returned to the flywheel as high-grade energy is*,

$$\text{A.E.} = 1.9 \times 4.18 \int_{293}^{295.67} \left(1 - \frac{293}{T}\right) dT$$

$$= 1.9 \times 4.18 \left[(295.67 - 293) - 293 \log_e \left(\frac{295.67}{293}\right)\right] = \mathbf{0.096\ kJ.}\quad \textbf{(Ans.)}$$

The amount of energy rendered unavailable is

$$\text{U.E.} = (\text{A.E.})_{\text{initial}} - (\text{A.E.})_{\text{returnable as high grade energy}}$$

$$= 21.2 - 0.096 = 21.1 \text{ kJ.}$$

Since the amount of energy returnable to the flywheel is 0.096 kJ, if ω_2 is the final angular velocity, and the flywheel is set in motion with this energy, then

$$0.096 \times 10^3 = \frac{1}{2} \times 0.62 \times \omega_2^2$$

$$\therefore \quad \omega_2^2 = \frac{0.096 \times 10^3 \times 2}{0.62} = 309.67 \quad \text{or} \quad \omega_2 = 17.59 \text{ rad/s.}$$

If N_2 is the final r.p.m. of the flywheel, then

$$\omega_2 = \frac{2\pi N_2}{60} \quad \text{or} \quad N_2 = \frac{17.59 \times 60}{2\pi} = 168 \text{ r.p.m.}$$

i.e., **Final r.p.m. of the flywheel = 168 r.p.m. (Ans.)**

☞**Example 6.21.** *The air, in a steady flow, enters the system at a pressure of 8 bar and 180°C with a velocity of 80 m/s and leaves at 1.4 bar and 20°C with a velocity of 40 m/s. The temperature of the surroundings is 20°C and pressure is 1 bar. Determine :*

(*i*) *Reversible work and actual work assuming the process to be adiabatic ;*

(*ii*) *Irreversibility and effectiveness of the system on the basis of 1 kg of air flow.*

Take for air : c_p *= 1.005 kJ/kg K ; R = 0.287 kJ/kg K.*

Solution. Initial pressure of air, p_1 = 8 bar

Initial temperature of air, T_1 = 180 + 273 = 453 K

Final pressure of air, p_2 = 1.4 bar

Final temperature, $T_2 = T_0$ = 20 + 273 = 293 K

Surroundings' pressure, p_0 = 1 bar

Mass of air = 1 kg

Initial velocity of air, C_1 = 80 m/s

Final velocity of air, C_2 = 40 m/s.

(*i*) **Reversible work and actual work :**

Availability of air at the inlet

$$= (h_1 - h_0) - T_0 (s_1 - s_0) + \frac{C_1^2}{2}$$

$$= c_p (T_1 - T_0) - T_0 (s_1 - s_0) + \frac{C_1^2}{2}$$

$$(s_1 - s_0) = c_p \log_e \left(\frac{T_1}{T_0}\right) - R \log_e \left(\frac{p_1}{p_0}\right)$$

$$= 1.005 \log_e \left(\frac{453}{293}\right) - 0.287 \log_e \left(\frac{8}{1}\right)$$

$$= 0.437 - 0.596 = -0.159 \text{ kJ/kg K}$$

∴ Availability of air at the inlet

$$= 1.005 (453 - 293) - 293 (-0.159) + \frac{80^2}{2 \times 10^3}$$

$$= 160.8 + 46.58 + 3.2 = 210.58 \text{ kJ}$$

Availability at the exit

$$= (h_2 - h_0) - T_0 (s_2 - s_0) + \frac{C_2^2}{2}$$

$$= -T_0 (s_2 - s_0) + \frac{C_2^2}{2}$$

as $h_2 = h_0$ because $T_2 = T_0$ = 293 K

Now $$s_2 - s_0 = -R \log_e \left(\frac{p_2}{p_0}\right) = -0.287 \log_e \left(\frac{1.4}{1}\right) = -0.09656 \text{ kJ/kg K}$$

∴ Availability at the exit

$$= -293 (-0.09656) + \frac{40^2}{2 \times 10^3} = 29.09 \text{ kJ/kg}$$

Reversible/theoretical work which must be available,

$$W_{rev} = 210.58 - 29.09 = \mathbf{181.49\ kJ.\ (Ans.)}$$

Actual work developed can be calculated by using the energy equation for adiabatic steady flow process as follows :

$$W_{actual} = (h_1 - h_2) + \left(\frac{C_1^2 - C_2^2}{2}\right)$$

$$= c_p\,(T_1 - T_2) + \left(\frac{C_1^2 - C_2^2}{2}\right)$$

$$= 1.005(453 - 293) + \left(\frac{80^2 - 40^2}{2 \times 10^3}\right)$$

$$= 160.8 + 2.4 = \mathbf{163.2\ kJ/kg.\quad (Ans.)}$$

(*ii*) **Irreversibility and effectiveness :**

Irreversibility, $I = W_{rev} - W_{actual}$

$$= 181.49 - 163.2 = \mathbf{18.29\ kJ/kg.\quad (Ans.)}$$

Effectiveness, $\epsilon = \dfrac{W_{actual}}{W_{rev}} = \dfrac{163.2}{181.49}$

$$= \mathbf{0.899} \text{ or } \mathbf{89.9\%}.\quad \mathbf{(Ans.)}$$

Example 6.22. *Steam expands adiabatically in a turbine from 20 bar, 400°C to 4 bar, 250°C. Calculate :*

(*i*) *The isentropic efficiency of the process ;*

(*ii*) *The loss of availability of the system assuming an atmospheric temperature of 20°C ;*

(*iii*) *The effectiveness of the process ;*

The changes in K.E. and P.E. may be neglected.

Solution. Initial pressure of steam, p_1 = 20 bar

Initial temperature of steam, t_1 = 400°C

Final pressure of steam, p_2 = 4 bar

Final temperature of steam, t_2 = 250°C

Atmospheric temperature, = 20°C (= 293 K).

Initial state 1 : 20 bar, 400°C ; From steam tables,

$$h_1 = 3247.6 \text{ kJ/kg} \ ; \ s_1 = 7.127 \text{ kJ/kg K}$$

Final state 2 : 4 bar 250°C ; From steam tables,

$$h_2' = 2964.2 \text{ kJ/kg}, \ s_2' = 7.379 \text{ kJ/kg K}$$

The process is shown as 1 to 2′ in Fig. 6.14

$$s_1 = s_2 = 7.127 \text{ kJ/kg K}$$

Hence, interpolating,

$$h_2 = 2752.8 + \left(\frac{7.127 - 6.930}{7.171 - 6.930}\right)(2860.5 - 2752.8)$$

$$= 2752.8 + \frac{0.197}{0.241} \times 107.7 = 2840.8 \text{ kJ/kg.}$$

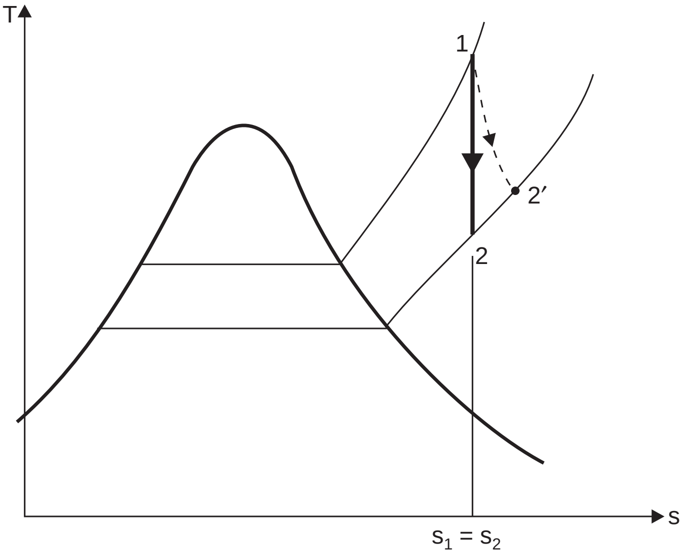


Fig. 6.14

(*i*) **Isentropic efficiency :**

$$\eta_{isen.} = \frac{\text{Actual work output}}{\text{Isentropic work}}$$

$$= \frac{h_1 - h_2'}{h_1 - h_2} = \frac{3247.6 - 2964.2}{3247.6 - 2840.8}$$

$$= \frac{283.4}{406.8} = \mathbf{0.6966} \text{ or } \mathbf{69.66\%}. \quad \textbf{(Ans.)}$$

(*ii*) **Loss of availability :**

Loss of availability $= b_1 - b_2$

$= h_1 - h_2' + T_0 (s_2' - s_1')$

$= 3247.6 - 2964.2 + 293 (7.379 - 7.127)$

$= 283.4 + 73.83 =$ **357.23 kJ/kg. (Ans.)**

(*iii*) **Effectiveness :**

Effectiveness, $\in = \frac{W}{b_1 - b_2'} = \frac{h_1 - h_2'}{b_1 - b_2'}$

$$= \frac{3247.6 - 2964.2}{357.9} = \mathbf{0.7918 \text{ or } 79.18\%}. \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. '*Available energy*' is the maximum portion of the energy which could be converted into useful work by ideal processes which reduce the system to a dead state.
2. The theoretical maximum amount of work which can be obtained from a system at any state p_1 and T_1 when operating with a reservoir at the constant pressure and temperature p_0 and T_0 is called '*availability*'.

3. Energy is said to be *degraded* each time it flows through a finite temperature difference. That is, why the second law of thermodynamics is sometimes called the *law of the degradation of energy*, and energy is said to '*run down hill*'.
4. In non-flow systems :
   Maximum work available,
   $$W_{max} = (u_1 - u_0) - T_0(s_1 - s_0) - p_0(v_0 - v_1)$$
   $$= (u_1 + p_0v_1 - T_0s_1) - (u_0 + p_0v_0 - T_0s_0)$$
   $$= a_1 - a_0 \quad \text{...per unit mass}$$
   The property $a = u + p_0v - T_0s$ is called the *non-flow availability function*.
5. In steady-flow systems :
   Maximum work available,
   $$W_{max} = (h_1 - T_0s_1) - (h_0 - T_0s_0)$$
   $$= b - b_0 \quad \text{...per unit mass}$$
   The property, $b = h - T_0s$ is called the *steady-flow availability function*.
6. It may be noted that Gibb's function $g = (h - Ts)$ is a property of the system where availability function $a = u + p_0v - T_0s$ is a composite property of the system and surroundings.
   Again, $a = u + p_0v - T_0s$
   $b = u + pv - T_0s$
   $g = u + pv - Ts$
   When state 1 proceeds to dead state (zero state)
   $a = b = g.$
7. The actual work which a system does is always less than the idealized reversible work, and the difference between the two is called the *irreversibility of the process*. This is also sometimes referred to as *degradation* or *dissipation*.
   Effectiveness is defined as the ratio of actual useful work to the maximum useful work.

OBJECTIVE TYPE QUESTIONS

Choose the correct answer :

1. Gibb's function is expressed as
   (*a*) $(u + pv - Ts)$ (*b*) $(u + pv - Tds)$
   (*c*) $(u + pdv - Tds)$ (*d*) $(u + pv - sdT)$.
2. Availability function is expressed as
   (*a*) $a = (u + p_0v - T_0s)$ (*b*) $a = (u + p_0dv + T_0ds)$
   (*c*) $a = (du + p_0dv - T_0ds)$ (*d*) $a = (u + p_0v + T_0s)$.
3. To increase work capacity of energy transferred by heat transfer from high temperature to low temperature
   (*a*) lower temperature should be lowered keeping temperature difference same
   (*b*) higher temperature should be increased keeping temperature difference same
   (*c*) temperature difference should be increased (*d*) temperature difference should be decreased.
4. Helmholtz function is expressed as
   (*a*) $(u - Ts)$ (*b*) $(h - Ts)$
   (*c*) $(- sdT + vdp)$ (*d*) $(u + pv)$.
5. If a heat source at temperature T_1 transfers heat to a system at temperature $T_2 (T_1 > T_2)$, state which of the following statements is *not true* ?
   (*a*) Δ_{system} decreases (*b*) Δ_{source} decreases
   (*c*) $(\Delta_{system} + \Delta_{source})$ decreases (*d*) $(\Delta_{system} + \Delta_{source})$ increases.

Answers

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*a*) **5.** (*d*).

THEORETICAL QUESTIONS

1. Explain the concept of available and unavailable energy. When does the system become dead ?
2. Define the term 'availability'.
3. Is the availability function same for a non-flow and a flow process ?
4. Define availability function and find the relationship between availability function and change in availability.
5. How are the concepts of entropy and unavailable energy related to each other ?
6. Derive an expression for availability in non-flow systems.
7. Derive an expression for availability in steady flow systems.
8. Differentiate between availability function and Gibb's energy function.
9. Derive an expression for decrease in available energy when heat is transferred through a finite temperature difference.
10. Derive a general expression for irreversibility in (*i*) non-flow process, (*ii*) steady flow process.
11. What is the effectiveness of a system and how does it differ from efficiency ?

UNSOLVED EXAMPLES

1. A system receives 10000 kJ of heat at 500 K from a source at 1000 K. The temperature of the surroundings is 300 K. Assuming that the temperature of the system and source remains constant during heat transfer, find :
   (*i*) The entropy production due to above mentioned heat transfer ;
   (*ii*) Decrease in available energy. [**Ans.** (*i*) 10 kJ/K ; (*ii*) 3000 kJ]
2. In a power station, saturated steam is generated at 252°C by transferring heat from the hot gases generated in the combustion chamber. The gases are cooled from 1100°C to 550°C during transferring the heat for steam generation. Determine the increase in total entropy of the combined system of gas and steam and increase in unavailable energy on the basis of one kg of steam generated. Assume water enters the boiler at saturated condition and leaves as saturated steam. [**Ans.** 1.99 kJ/K ; 597 kJ/kg of steam formed]
3. Air at 15°C is to be heated to 40°C by mixing it in steady flow with a quantity of air at 90°C. Assuming that the mixing process is adiabatic and neglecting changes in kinetic and potential energy, calculate the ratio of the mass flow of air initially at 90°C to that initially at 15°C. Calculate also the effectiveness of the heating process, if the atmospheric temperature is 15°C. [**Ans.** 0.5, 0.327 or 32.7%]
4. A liquid of specific heat 6.3 kJ/kg K is heated at approximately constant pressure from 15°C to 70°C by passing it through tubes which are immersed in a furnace. The furnace temperature is constant at 1400°C. Calculate the effectiveness of the heating process when the atmospheric temperature is 10°C. [**Ans.** 0.121 or 12.1%]
5. 500 kJ of heat from an infinite source at 1000 K is supplied to 2 kg of gas initially at 2 bar and 350 K in a closed tank. Find the loss in available energy due to above heat transfer. Take : c_v (gas) = 0.8 kJ/kg K and surrounding temperature = 300 K. [**Ans.** 157.5 kJ]
6. In an heat exchanger of parallel flow type, water enters at 60°C and leaves at 80°C while oil of specific gravity 0.8 enters at 250°C and leaves at 100°C. The specific heat of oil is 2.5 kJ/kg K and surrounding temperature is 300 K. Determine the loss in availability on the basis of one kg of oil flow per second. [**Ans.** – 59.9 kJ]
7. 1 kg of ice at 0°C is mixed with 10 kg of water at 30°C. Determine the net increase in the entropy and unavailable energy when the system reaches common temperature. Assume that surrounding temperature is 10°C. Take, specific heat of water = 4.18 kJ/kg K ; specific heat of ice = 2.1 kJ/kg K ; latent heat of ice = 333.5 kJ/kg. [**Ans.** 0.114 kJ/K, 32.30 kJ]

8. 1000 kJ of heat leaves hot gases at 1400°C from a fire box of a boiler and goes to steam at 250°C. Atmospheric temperature is 20°C. Divide the energy into available and unavailable portions :

(*i*) As it leaves the hot gases.

(*ii*) As it enters the steam. [**Ans.** (*i*) 825 kJ, 175 kJ ; (*ii*) 440 kJ, 580 kJ]

9. In a certain process, a vapour, while condensing at 420°C, transfers heat to water evaporating at 250°C. The resulting steam is used in a power cycle which rejects heat at 35°C. What is the fraction of the available energy in the heat transferred from the process vapour at 420°C that is lost due to the irreversible heat transfer at 250°C. [**Ans.** 0.26]

10. In a steam boiler, hot gases from a fire transfer heat to water which vapourizes at constant temperature. In certain case, the gases are cooled from 1100°C to 550°C while the water evaporates at 220°C. The specific heat of gases is 1.005 kJ/kg K, and the latent heat of water at 220°C is 1858.5 kJ/kg. All the heat transferred from the gases goes to the water. How much does the total entropy of the combined system of gas and water increase as a result of irreversible heat transfer ? Obtain the result on the basis of 1 kg of water evaporated. If the temperature of the surroundings is 30°C find the increase in unavailable energy due to irreversible heat transfer. [**Ans.** 2.045 kJ/K, 620 kJ]

11. Calculate the unavailable energy in 40 kg of water at 75°C with respect to the surroundings at 5°C, the pressure being 1 atmosphere. [**Ans.** 10420 kJ]

12. Calculate the decrease in available energy when 25 kg of water at 95°C mixes with 35 kg of water at 35°C, the pressure being taken as constant and the temperature of the surroundings being 15°C.

Take c_p of water = 4.18 kJ/kg K. [**Ans.** 281.81 kJ]

13. 2 kg of air at 5 bar, 80°C expands adiabatically in a closed system until its volume is doubled and its temperature becomes equal to that of the surroundings which is at 1 bar, 5°C. For this process, determine :

(*i*) The maximum work ;

(*ii*) The change in availability ;

(*iii*) The irreversibility.

For air take : c_v = 0.718 kJ/kg K, $u = c_v T$, where c_v is constant and $pV = mRT$, where p is in bar, V volume in m^3, m mass in kg, R is constant equal to 0.287 kJ/kg K, and T temperature in K.

[**Ans.** (*i*) 122.72 kJ ; (*ii*) 82.2 kJ ; (*iii*) 15.2 kJ]

14. One kg of air at a pressure p, and temperature 900 K is mixed with 1 kg of air at the same pressure and 500 K.

Determine the loss in availability if the surrounding temperature is 300 K. [**Ans.** 54 kJ]

15. 10 kg of water is heated in an insulated tank by a churning process from 300 K to 350 K. Determine the loss in availability for the process if the surrounding temperature is 300 K. [**Ans.** 1968 kJ]

16. A closed system contains 10 kg of air at 600 K and 5 bar. Determine the availability of the system if the surrounding pressure and temperature are 1 bar and 300 K respectively.

If the air is cooled at constant pressure to the atmospheric temperature, determine the availability and effectiveness. [**Ans.** 1464 kJ ; 921 kJ ; 0.63]

17. In a turbine air expands from 5 bar, 520°C to 1 bar, 300°C. During expansion 10 kJ/kg of heat is lost to the surroundings which is at 0.98 bar, 20°C. Neglecting kinetic and potential energy changes, determine per kg of air :

(*i*) The decrease in availability ;

(*ii*) The maximum work ;

(*iii*) The irreversibility.

For air take : c_p = 1.005 kJ/kg K ; $h = c_p T$, where c_p is constant.

[**Ans.** (*i*) 260.7 kJ/kg ; (*ii*) 260.7 kJ/kg ; (*iii*) 49.6 kJ/kg]

18. A centrifugal air compressor compresses air at the rate of 20 kg/min from 1 bar to 2 bar. The temperature increases from 20°C to 120°C during the compression. Determine actual and minimum power required to run the compressor. The surrounding air temperature is 20°C.

Neglect the heat interaction between the compressor and surroundings and changes in potential and kinetic energy. [**Ans.** 24.2 kW]

19. 1 kg of air is compressed polytropically from 1 bar and 300 K to 7 bar and 380 K. Determine the irreversibility and effectiveness, assuming temperature and pressure as 300 K and 1 bar.

Take for air : c_p = 1.005 kJ/kg K, c_p = 0.718 kJ/kg K, R = 0.287 kJ/kg K. [**Ans.** 1.25 kJ ; 0.924]

20. The moment of inertia of a flywheel is 0.54 kg-m^2 and it rotates at a speed of 3000 r.p.m. in a large heat insulated system, the temperature of which is 15°C. If the kinetic energy of the flywheel is dissipated as frictional heat at the shaft bearings which have a water equivalent of 2 kg, find the rise in the temperature of the bearings when the flywheel has come to rest.

Calculate the greatest possible amount of the above heat which may be returned to the flywheel as high-grade energy, showing how much of the original kinetic energy is now unavailable. What would be the final r.p.m. of the flywheel, if it is set in motion with this available energy ?

[**Ans.** 3.19°C ; 0.1459 kJ ; 222 r.p.m.]

21. In a steady flow air enters the system at a pressure of 10 bar and 200°C with a velocity of 100 m/s and leaves at 1.5 bar and 25°C with a velocity of 50 m/s. The temperature of the surroundings is 25°C and pressure is 1 bar. Determine reversible work and actual work assuming the process to be adiabatic.

Determine also the irreversibility and effectiveness of the system on the basis of one kg of air flow.

Take for air : c_p = 1 kJ/kg K ; R = 287 J/kg K. [**Ans.** 200.65 kJ ; 178.75 kJ/kg, 21.90 kJ/kg ; 0.894]