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Advanced Engineering Mathematics



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To Our Parents

Bhagat Ram Jain and Sampati Devi Jain

&

S.T.V. Raghavacharya and Rajya Lakshmi

whose memories had always been an inspiration

Preface

This book is based on the experience and the lecture notes of the authors while teaching mathematics courses to engineering students at the Indian Institute of Technology, Delhi for more than three decades. A number of available textbooks have been a source of inspiration for introduction of concepts and formulation of problems. We are thankful to the authors of these books for their indirect help.

This comprehensive textbook covers syllabus for two courses in Mathematics for engineering students in various Institutes, Universities and Engineering Colleges. The emphasis is on the presentation of the fundamentals and theoretical concepts in an intelligible and easy to understand manner.

Each chapter in the book has been carefully planned to make it an effective tool to arouse interest in the study and application of mathematics to solve engineering and scientific problems. Simple and illustrative examples are used to explain each theoretical concept. Graded sets of examples and exercises are given in each chapter, which will help the students to understand every important concept. The book contains 682 solved examples and 2984 problems in the exercises. Answers to every problem and hints for difficult problems are given at the end of each chapter which will motivate the students for self-learning. While some problems emphasize the theoretical concepts, others provide enough practice and generate confidence to use these concepts in problem solving. This textbook offers a logical and lucid presentation of both the theory and problem solving techniques so that the student is not lost in unnecessary details.

We hope that this textbook will meet the requirements and the expectations of all the engineering students.

We will gratefully receive and acknowledge every comment, suggestions for inclusion/exclusion of topics and errors in the book, both from the faculty and the students.

We are grateful to our former teachers, colleagues and well wishers for their encouragement and valuable suggestions. We are also thankful to our students for their feed back. We are grateful to the authorities of IIT Delhi for providing us their support.

We extend our thanks to the editorial and the production staff of M/s Narosa Publishing House, in particular Mr. Mohinder Singh Sejwal, for their care and enthusiasm in the preparation of this book.

Last, but not the least, we owe a lot to our family members, in particular, our wives Vinod Jain and Seetha Lakshmi whose encouragement and support had always been inspiring and rejuvenating. We appreciate their patience during our long hours of work day and night.

New Delhi
October 2001

R.K. JAIN
S.R.K. IYENGAR

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