

Eleventh Edition

Vector Mechanics For Engineers

Statics and Dynamics

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VECTOR MECHANICS FOR ENGINEERS: STATICS AND DYNAMICS, ELEVENTH EDITION

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Preface

Objectives

A primary objective in a first course in mechanics is to help develop a student's ability first to analyze problems in a simple and logical manner, and then to apply basic principles to their solutions. A strong conceptual understanding of these basic mechanics principles is essential for successfully solving mechanics problems. We hope this text will help instructors achieve these goals.

General Approach

Vector algebra was introduced at the beginning of the first volume and is used in the presentation of the basic principles of statics, as well as in the solution of many problems, particularly three-dimensional problems. Similarly, the concept of vector differentiation will be introduced early in this volume, and vector analysis will be used throughout the presentation of dynamics. This approach leads to more concise derivations of the fundamental principles of mechanics. It also makes it possible to analyze many problems in kinematics and kinetics which could not be solved by scalar methods. The emphasis in this text, however, remains on the correct understanding of the principles of mechanics and on their application to the solution of engineering problems, and vector analysis is presented chiefly as a convenient tool.[†]

Practical Applications Are Introduced Early. One of the characteristics of the approach used in this book is that mechanics of *particles* is clearly separated from the mechanics of *rigid bodies*. This approach makes it possible to consider simple practical applications at an early stage and to postpone the introduction of the more difficult concepts. For example:

- In *Statics*, the statics of particles is treated first, and the principle of equilibrium of a particle was immediately applied to practical situations involving only concurrent forces. The statics of rigid bodies is considered later, at which time the vector and scalar products of two vectors were introduced and used to define the moment of a force about a point and about an axis.
- In *Dynamics*, the same division is observed. The basic concepts of force, mass, and acceleration, of work and energy, and of impulse and momentum are introduced and first applied to problems involving only particles. Thus, students can familiarize themselves with the three basic methods used in dynamics and learn their respective advantages before facing the difficulties associated with the motion of rigid bodies.

[†]In a parallel text, *Mechanics for Engineers*, fifth edition, the use of vector algebra is limited to the addition and subtraction of vectors, and vector differentiation is omitted.

NEW!

The 11th edition has undergone a complete rewrite to modernize and streamline the language throughout the text.

2.2 ADDING FORCES BY COMPONENTS

In Sec. 2.1E, we described how to resolve a force into components. Here we discuss how to add forces by using their components, especially rectangular components. This method is often the most convenient way to add forces and, in practice, is the most common approach. (Note that we can readily extend the properties of vectors established in this section to the rectangular components of any vector quantity, such as velocity or momentum.)

2.2A Rectangular Components of a Force: Unit Vectors

In many problems, it is useful to resolve a force into two components that are perpendicular to each other. Figure 2.14 shows a force \mathbf{F} resolved into a component \mathbf{F}_x along the x axis and a component \mathbf{F}_y along the y axis. The parallelogram drawn to obtain the two components is a rectangle, and \mathbf{F}_x and \mathbf{F}_y are called **rectangular components**.

The x and y axes are usually chosen to be horizontal and vertical, respectively, as in Fig. 2.14; they may, however, be chosen in any two perpendicular directions, as shown in Fig. 2.15. In determining the

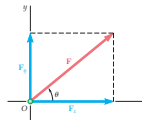


Fig. 2.14 Rectangular components of a force \mathbf{F} .

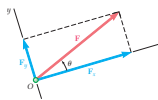


Fig. 2.15 Rectangular components of a force \mathbf{F} for axes rotated away from horizontal and vertical.

17.1 ENERGY METHODS FOR A RIGID BODY

We now use the principle of work and energy to analyze the plane motion of rigid bodies. As we pointed out in Chap. 13, the method of work and energy is particularly well adapted to solving problems involving velocities and displacements. Its main advantage is that the work of forces and the kinetic energy of particles are scalar quantities.

17.1A Principle of Work and Energy

To apply the principle of work and energy to the motion of a rigid body, we again assume that the rigid body is made up of a large number n of particles of mass Δm_i . From Eq. (14.30) of Sec. 14.2B, we have

Principle of work and energy, rigid body

$$T_1 + U_{1 \rightarrow 2} = T_2 \quad (17.1)$$

where T_1, T_2 = the initial and final values of total kinetic energy of particles forming the rigid body

$U_{1 \rightarrow 2}$ = work of all forces acting on various particles of the body

Just as we did in Chap. 13, we can express the work done by nonconservative forces as $U_{1 \rightarrow 2}^{NC}$, and we can define potential energy terms for conservative forces. Then we can express Eq. (17.1) as

$$T_1 + V_{g_1} + V_{e_1} + U_{1 \rightarrow 2}^{NC} = T_2 + V_{g_2} + V_{e_2} \quad (17.1')$$

where V_{g_1} and V_{g_2} are the initial and final gravitational potential energy of the center of mass of the rigid body with respect to a reference point or datum, and V_{e_1} and V_{e_2} are the initial and final values of the elastic energy associated with springs in the system.

We obtain the total kinetic energy

$$T = \frac{1}{2} \sum_{i=1}^n \Delta m_i v_i^2 \quad (17.2)$$

by adding positive scalar quantities, so it is itself a positive scalar quantity. You will see later how to determine T for various types of motion of a rigid body.

The expression $U_{1 \rightarrow 2}$ in Eq. (17.1) represents the work of all the forces acting on the various particles of the body whether these forces are internal or external. However, the total work of the internal forces holding together the particles of a rigid body is zero. To see this, consider two particles A and B of a rigid body and the two equal and opposite forces \mathbf{F} and $-\mathbf{F}$ they exert on each other (Fig. 17.1). Although, in general, small displacements $d\mathbf{r}$ and $d\mathbf{r}'$ of the two particles are different, the components of these displacements along AB must be equal; otherwise, the particles would not remain at the same distance from each other and the body would not be rigid. Therefore, the work of \mathbf{F} is equal in magnitude and

New Concepts Are Introduced in Simple Terms. Since this text is designed for the first course in dynamics, new concepts are presented in simple terms and every step is explained in detail. On the other hand, by discussing the broader aspects of the problems considered, and by stressing methods of general applicability, a definite maturity of approach has been achieved. For example, the concept of potential energy is discussed in the general case of a conservative force. Also, the study of the plane motion of rigid bodies is designed to lead naturally to the study of their general motion in space. This is true in kinematics as well as in kinetics, where the principle of equivalence of external and effective forces is applied directly to the analysis of plane motion, thus facilitating the transition to the study of three-dimensional motion.

Fundamental Principles Are Placed in the Context of Simple Applications. The fact that mechanics is essentially a *deductive* science based on a few fundamental principles is stressed. Derivations have been presented in their logical sequence and with all the rigor warranted at this level. However, the learning process being largely *inductive*, simple applications are considered first. For example:

- The statics of particles precedes the statics of rigid bodies, and problems involving internal forces are postponed until Chap. 6.
- In Chap. 4, equilibrium problems involving only coplanar forces are considered first and solved by ordinary algebra, while problems involving three-dimensional forces and requiring the full use of vector algebra are discussed in the second part of the chapter.
- The kinematics of particles (Chap. 11) precedes the kinematics of rigid bodies (Chap. 15).
- The fundamental principles of the kinetics of rigid bodies are first applied to the solution of two-dimensional problems (Chaps. 16 and 17), which can be more easily visualized by the student, while three-dimensional problems are postponed until Chap. 18.

The Presentation of the Principles of Kinetics Is Unified.

The eleventh edition of *Vector Mechanics for Engineers* retains the unified presentation of the principles of kinetics which characterized the previous ten editions. The concepts of linear and angular momentum are introduced in Chap. 12 so that Newton's second law of motion can be presented not only in its conventional form $\mathbf{F} = m\mathbf{a}$, but also as a law relating, respectively, the sum of the forces acting on a particle and the sum of their moments to the rates of change of the linear and angular momentum of the particle. This makes possible an earlier introduction of the principle of conservation of angular momentum and a more meaningful discussion of the motion of a particle under a central force (Sec. 12.3A). More importantly, this approach can be readily extended to the study of the motion of a system of particles (Chap. 14) and leads to a more concise and unified treatment of the kinetics of rigid bodies in two and three dimensions (Chaps. 16 through 18).

Systematic Problem-Solving Approach. New to this edition of the text, all the sample problems are solved using the steps of Strategy, Modeling, Analysis, and Reflect & Think, or the "SMART" approach.

NEW!

This methodology is intended to give students confidence when approaching new problems, and students are encouraged to apply this approach in the solution of all assigned problems.

Free-Body Diagrams Are Used Both to Solve Equilibrium Problems and to Express the Equivalence of Force Systems. Free-body diagrams were introduced early in statics, and their importance was emphasized throughout. They were used not only to solve equilibrium problems but also to express the equivalence of two systems of forces or, more generally, of two systems of vectors. In dynamics we will introduce a kinetic diagram, which is a pictorial representation of inertia terms. The advantage of this approach becomes apparent in the study of the dynamics of rigid bodies, where it is used to solve three-dimensional as well as two-dimensional problems. By placing the emphasis on the free-body diagram and kinetic diagram, rather than on the standard algebraic equations of motion, a more intuitive and more complete understanding of the fundamental principles of dynamics can be achieved. This approach, which was first introduced in 1962 in the first edition of *Vector Mechanics for Engineers*, has now gained wide acceptance among mechanics teachers in this country. It is, therefore, used in preference to the method of dynamic equilibrium and to the equations of motion in the solution of all sample problems in this book.

A Careful Balance between SI and U.S. Customary Units Is Consistently Maintained. Because of the current trend in the American government and industry to adopt the international system of units (SI metric units), the SI units most frequently used in mechanics are introduced in Chap. 1 and are used throughout the text. Approximately half of the sample problems and 60 percent of the homework problems are stated in these units, while the remainder are in U.S. customary units. The authors believe that this approach will best serve the need of the students, who, as engineers, will have to be conversant with both systems of units.

It also should be recognized that using both SI and U.S. customary units entails more than the use of conversion factors. Since the SI system of units is an absolute system based on the units of time, length, and mass, whereas the U.S. customary system is a gravitational system based on the units of time, length, and force, different approaches are required for the solution of many problems. For example, when SI units are used, a body is generally specified by its mass expressed in kilograms; in most problems of statics it will be necessary to determine the weight of the body in newtons, and an additional calculation will be required for this purpose. On the other hand, when U.S. customary units are used, a body is specified by its weight in pounds and, in dynamics problems, an additional calculation will be required to determine its mass in slugs (or $\text{lb}\cdot\text{s}^2/\text{ft}$). The authors, therefore, believe that problem assignments should include both systems of units.

The *Instructor's and Solutions Manual* provides six different lists of assignments so that an equal number of problems stated in SI units and in U.S. customary units can be selected. If so desired, two complete lists of assignments can also be selected with up to 75 percent of the problems stated in SI units.

Sample Problem 3.10

Three cables are attached to a bracket as shown. Replace the forces exerted by the cables with an equivalent force-couple system at A.

STRATEGY: First determine the relative position vectors drawn from point A to the points of application of the various forces and resolve the forces into rectangular components. Then sum the forces and moments.

MODELING and ANALYSIS: Note that $F_B = (700 \text{ N})\mathbf{A}_{60}$, where

$$\mathbf{A}_{60} = \frac{\mathbf{BC}}{BC} = \frac{75\mathbf{i} + 150\mathbf{j} + 50\mathbf{k}}{175}$$

Using meters and newtons, the position and force vectors are

$$\begin{aligned} \mathbf{r}_{AB} &= \mathbf{AB} = 0.075\mathbf{i} + 0.050\mathbf{k} & \mathbf{F}_A &= 300\mathbf{i} - 600\mathbf{j} + 200\mathbf{k} \\ \mathbf{r}_{AC} &= \mathbf{AC} = 0.075\mathbf{i} - 0.050\mathbf{k} & \mathbf{F}_C &= 707\mathbf{i} & & - 707\mathbf{k} \\ \mathbf{r}_{BC} &= \mathbf{BC} = 0.100\mathbf{i} - 0.100\mathbf{j} & \mathbf{F}_B &= 608\mathbf{i} + 1039\mathbf{j} \end{aligned}$$

The force-couple system at A equivalent to the given forces consists of a force $\mathbf{R} = \Sigma \mathbf{F}$ and a couple $\mathbf{M}_A^O = \Sigma (\mathbf{r} \times \mathbf{F})$. Obtain the force \mathbf{R} by adding respectively the x , y , and z components of the forces:

$$\mathbf{R} = \Sigma \mathbf{F} = (1607 \text{ N}\mathbf{i} + 439 \text{ N}\mathbf{j} - 507 \text{ N}\mathbf{k}) \quad \leftarrow$$

(continued)

Remark: Since all the forces are contained in the plane of the figure, you would expect the sum of their moments to be perpendicular to that plane. Note that you could obtain the moment of each force component directly from the diagram by first forming the product of its magnitude and perpendicular distance to O and then assigning to this product a positive or a negative sign, depending upon the sense of the moment.

d. Single Tugboat. The force exerted by a single tugboat must be equal to \mathbf{R} , and its point of application A must be such that the moment of \mathbf{R} about O is equal to \mathbf{M}_A^O (Fig. 3). Observing that the position vector of A is

$$\mathbf{r} = x\mathbf{i} + 70\mathbf{j}$$

you have

$$\begin{aligned} \mathbf{r} \times \mathbf{R} &= \mathbf{M}_A^O \\ (x\mathbf{i} + 70\mathbf{j}) \times (9.04\mathbf{i} - 9.79\mathbf{j}) &= -1035\mathbf{k} \\ -x(9.79)\mathbf{k} - 633\mathbf{k} &= -1035\mathbf{k} & x &= 41.1 \text{ ft} \quad \leftarrow \end{aligned}$$

REFLECT and THINK: Reducing the given situation to that of a single force makes it easier to visualize the overall effect of the tugboats in maneuvering the ocean liner. But in practical terms, having four boats applying force allows for greater control in slowing and turning a large ship in a crowded harbor.

Optional Sections Offer Advanced or Specialty Topics.

A large number of optional sections have been included. These sections are indicated by asterisks and thus are easily distinguished from those which form the core of the basic dynamics course. They can be omitted without prejudice to the understanding of the rest of the text.

The topics covered in the optional sections in statics include the reduction of a system of forces of a wrench, applications to hydrostatics, equilibrium of cables, products of inertia and Mohr's circle, the determination of the principal axes and the mass moments of inertia of a body of arbitrary shape, and the method of virtual work. The sections on the inertia properties of three-dimensional bodies are primarily intended for students who will later study in dynamics the three-dimensional motion of rigid bodies.

The topics covered in the optional sections in dynamics include graphical methods for the solution of rectilinear-motion problems, the trajectory of a particle under a central force, the deflection of fluid streams, problems involving jet and rocket propulsion, the kinematics and kinetics of rigid bodies in three dimensions, damped mechanical vibrations, and electrical analogues. These topics will be found of particular interest when dynamics is taught in the junior year.

The material presented in the text and most of the problems require no previous mathematical knowledge beyond algebra, trigonometry, elementary calculus, and the elements of vector algebra presented in Chaps. 2 and 3 of the volume on statics. However, special problems are included, which make use of a more advanced knowledge of calculus, and certain sections, such as Secs. 19.5A and 19.5B on damped vibrations, should be assigned only if students possess the proper mathematical background. In portions of the text using elementary calculus, a greater emphasis is placed on the correct understanding and application of the concepts of differentiation and integration, than on the nimble manipulation of mathematical formulas. In this connection, it should be mentioned that the determination of the centroids of composite areas precedes the calculation of centroids by integration, thus making it possible to establish the concept of moment of area firmly before introducing the use of integration.

Guided Tour

Chapter Introduction. Each chapter begins with a list of learning objectives and an outline that previews chapter topics. An introductory section describes the material to be covered in simple terms, and how it will be applied to the solution of engineering problems.

Chapter Lessons. The body of the text is divided into sections, each consisting of one or more sub-sections, several sample problems, and a large number of end-of-section problems for students to solve. Each section corresponds to a well-defined topic and generally can be covered in one lesson. In a number of cases, however, the instructor will find it desirable to devote more than one lesson to a given topic. *The Instructor's and Solutions Manual* contains suggestions on the coverage of each lesson.

Sample Problems. The Sample Problems are set up in much the same form that students will use when solving assigned problems, and they employ the SMART problem-solving methodology that students are encouraged to use in the solution of their assigned problems. They thus serve the double purpose of reinforcing the text and demonstrating the type of neat and orderly work that students should cultivate in their own solutions. In addition, in-problem references and captions have been added to the sample problem figures for contextual linkage to the step-by-step solution.

Concept Applications. Concept Applications are used within selected theory sections in the Statics volume to amplify certain topics, and they are designed to reinforce the specific material being presented and facilitate its understanding.

Solving Problems on Your Own. A section entitled *Solving Problems on Your Own* is included for each lesson, between the sample problems and the problems to be assigned. The purpose of these sections is to help students organize in their own minds the preceding theory of the text and the solution methods of the sample problems so that they can more successfully solve the homework problems. Also included in these sections are specific suggestions and strategies that will enable the students to more efficiently attack any assigned problems.

Homework Problem Sets. Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and to help students understand the principles of mechanics. The problems are grouped according to the portions of material they illustrate and, in general, are arranged in order of increasing difficulty. Problems requiring special attention are indicated by asterisks. Answers to 70 percent of the problems are given at the end of the book. Problems for which the answers are given are set in straight type in the text, while problems for which no answer is given are set in italic and red font color.



1

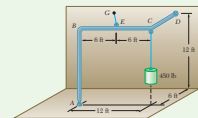
Introduction

The tallest skyscraper in the Western Hemisphere, One World Trade Center is a prominent feature of the New York City skyline. From its foundation to its structural components and mechanical systems, the design and operation of the tower is based on the fundamentals of engineering mechanics.

NEW!

Sample Problem 4.10

A 450-lb load hangs from the corner C of a rigid piece of pipe ABCD that has been bent as shown. The pipe is supported by ball-and-socket joints A and D, which are fastened, respectively, to the floor and to a vertical wall, and by a cable attached at the midpoint E of the portion BC of the pipe and at a point G on the wall. Determine (a) where G should be located if the tension in the cable is to be minimum, (b) the corresponding minimum value of the tension.



STRATEGY: Draw the free-body diagram of the pipe showing the reactions at A and D. Isolate the unknown tension T and the known weight W from the equilibrium equations.

MODELING and ANALYSIS:

Free-Body Diagram. The free-body diagram of the pipe includes the load $W = (-450 \text{ lb})\mathbf{j}$, the reactions at A and D, and the force T exerted by the cable (Fig. 1). To eliminate the reactions at A and D from the computations, take the sum of the moments of the forces about the line AD and set it equal to zero. Denote the unit vector along AD by λ , which enables you to write

$$\sum M_{AD} = 0: \lambda \cdot (\lambda E \times T) + \lambda \cdot (\lambda C \times W) = 0 \quad (1)$$

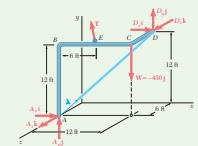


Fig. 1 Free-body diagram of pipe.

NEW!

Over 650 of the homework problems in the text are new or revised.

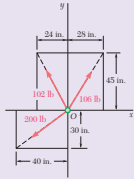
Chapter Review and Summary. Each chapter ends with a review and summary of the material covered in that chapter. Marginal notes are used to help students organize their review work, and cross-references have been included to help them find the portions of material requiring their special attention.

Review Problems. A set of review problems is included at the end of each chapter. These problems provide students further opportunity to apply the most important concepts introduced in the chapter.

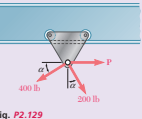
Review Problems

2.127 Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 15 kN in member *A* and 10 kN in member *B*, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members *A* and *B*.

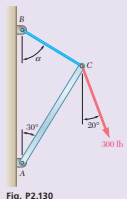
2.128 Determine the *x* and *y* components of each of the forces shown.



2.129 A hoist trolley is subjected to the three forces shown. Knowing that $\alpha = 40^\circ$, determine (a) the required magnitude of the force *P* if the resultant of the three forces is to be vertical, (b) the corresponding magnitude of the resultant.



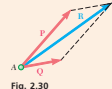
2.130 Knowing that $\alpha = 55^\circ$ and that boom *AC* exerts on pin *C* a force directed along line *AC*, determine (a) the magnitude of that force, (b) the tension in cable *BC*.



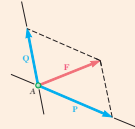
Review and Summary

In this chapter, we have studied the effect of forces on particles, i.e., on bodies of such shape and size that we may assume all forces acting on them apply at the same point.

Resultant of Two Forces
Forces are *vector quantities*; they are characterized by a point of application, a magnitude, and a direction, and they add according to the parallelogram law (Fig. 2.30). We can determine the magnitude and direction of the resultant *R* of two forces *P* and *Q* either graphically or by trigonometry using the law of cosines and the law of sines [Sample Prob. 2.1].



Components of a Force
Any given force acting on a particle can be resolved into two or more components, i.e., it can be replaced by two or more forces that have the same effect on the particle. A force *F* can be resolved into two components *P* and *Q* by drawing a parallelogram with *F* for its diagonal; the components *P* and *Q* are then represented by the two adjacent sides of the parallelogram (Fig. 2.31). Again, we can determine the components either graphically or by trigonometry [Sec. 2.1E].



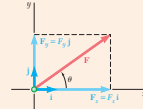
Rectangular Components; Unit Vectors
A force *F* is resolved into two rectangular components if its components *F_x* and *F_y* are perpendicular to each other and are directed along the coordinate axes (Fig. 2.32). Introducing the unit vectors *i* and *j* along the *x* and *y* axes, respectively, we can write the components and the vector as [Sec. 2.2A]

$$F_x = F_x i \quad F_y = F_y j \quad (2.6)$$

and

$$F = F_x i + F_y j \quad (2.7)$$

where *F_x* and *F_y* are the *scalar components* of *F*. These components, which can be positive or negative, are defined by the relations

$$F_x = F \cos \theta \quad F_y = F \sin \theta \quad (2.8)$$


Computer Problems. Accessible through Connect are problem sets for each chapter that are designed to be solved with computational software. Many of these problems are relevant to the design process; they may involve the analysis of a structure for various configurations and loadings of the structure, or the determination of the equilibrium positions of a given mechanism that may require an iterative method of solution. Developing the algorithm required to solve a given mechanics problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply their computer skills to the solution of a meaningful engineering problem.

Concept Questions. Educational research has shown that students can often choose appropriate equations and solve algorithmic problems without having a strong conceptual understanding of mechanics principles.[†] To help assess and develop student conceptual understanding, we have included Concept Questions, which are multiple choice problems that require few, if any, calculations. Each possible incorrect answer typically represents a common misconception (e.g., students often think that a vehicle moving in a curved path at constant speed has zero acceleration). Students are encouraged to solve these problems using the principles and techniques discussed in the text and to use these principles to help them develop their intuition. Mastery and discussion of these Concept Questions will deepen students' conceptual understanding and help them to solve dynamics problems.

Free Body and Impulse-Momentum Diagram Practice Problems. Drawing diagrams correctly is a critical step in solving kinetics problems in dynamics. A new type of problem has been added to the text to emphasize the importance of drawing these diagrams. In Chaps. 12 and 16 the Free Body Practice Problems require students to draw a free-body diagram (FBD) showing the applied forces and an equivalent diagram called a "kinetic diagram" (KD) showing $m\mathbf{a}$ or its components and $\bar{I}\boldsymbol{\alpha}$. These diagrams provide students with a pictorial representation of Newton's second law and are critical in helping students to correctly solve kinetic problems. In Chaps. 13 and 17 the Impulse-Momentum Diagram Practice Problems require students to draw diagrams showing the momenta of the bodies before impact, the impulses exerted on the body during impact, and the final momenta of the bodies. The answers to all of these questions can be accessed through Connect.

FREE-BODY PRACTICE PROBLEMS

16.F1 A 6-ft board is placed in a truck with one end resting against a block secured to the floor and the other leaning against a vertical partition. Draw the FBD and KD necessary to determine the maximum allowable acceleration of the truck if the board is to remain in the position shown.

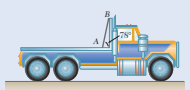


Fig. P16.F1

16.F2 A uniform circular plate of mass 3 kg is attached to two links AC and BD of the same length. Knowing that the plate is released from rest in the position shown, in which lines joining C to A and B to D are, respectively, horizontal and vertical, draw the FBD and KD for the plate.

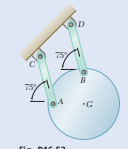


Fig. P16.F2

16.F3 Two uniform disks and two cylinders are assembled as indicated. Disk A weighs 20 lb and disk B weighs 12 lb. Knowing that the system is released from rest, draw the FBD and KD for the whole system.

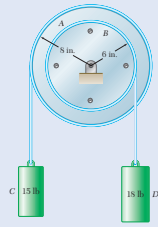


Fig. P16.F3

16.F4 The 400-lb crate shown is lowered by means of two overhead cranes. Knowing the tension in each cable, draw the FBD and KD that can be used to determine the angular acceleration of the crate and the acceleration of the center of gravity.

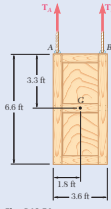


Fig. P16.F4

[†]Hestenes, D., Wells, M., and Swakhamer, G (1992). The force concept inventory. *The Physics Teacher*, 30: 141–158.

Streveler, R. A., Litzinger, T. A., Miller, R. L., and Steif, P. S. (2008). Learning conceptual knowledge in the engineering sciences: Overview and future research directions, *JEE*, 279–294.

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Find the following instructor resources available through Connect:

- **Instructor's and Solutions Manual.** *The Instructor's and Solutions Manual* that accompanies the eleventh edition features solutions to all end of chapter problems. This manual also features a number of tables designed to assist instructors in creating a schedule of assignments for their course. The various topics covered in the text have been listed in Table I and a suggested number of periods to be spent on each topic has been indicated. Table II prepares a brief description of all groups of problems and a classification of the problems in each group according to the units used. Sample lesson schedules are shown in Tables III, IV, and V, together with various alternative lists of assigned homework problems.
- **Lecture PowerPoint Slides** for each chapter that can be modified. These generally have an introductory application slide, animated worked-out problems that you can do in class with your students, concept questions, and “what-if?” questions at the end of the units.
- **Textbook images**
- **Computer Problem sets** for each chapter that are designed to be solved with computational software.
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List of Symbols

\mathbf{a}, a	Acceleration	\bar{k}	Centroidal radius of gyration
a	Constant; radius; distance; semimajor axis of ellipse	l	Length
$\bar{\mathbf{a}}, \bar{a}$	Acceleration of mass center	\mathbf{L}	Linear momentum
$\mathbf{a}_{B/A}$	Acceleration of B relative to frame in translation with A	L	Length; inductance
$\mathbf{a}_{P/\mathcal{F}}$	Acceleration of P relative to rotating frame \mathcal{F}	m	Mass
\mathbf{a}_c	Coriolis acceleration	m'	Mass per unit length
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$	Reactions at supports and connections	\mathbf{M}	Couple; moment
A, B, C, \dots	Points	\mathbf{M}_O	Moment about point O
A	Area	\mathbf{M}_O^R	Moment resultant about point O
b	Width; distance; semiminor axis of ellipse	M	Magnitude of couple or moment; mass of earth
c	Constant; coefficient of viscous damping	M_{OL}	Moment about axis OL
C	Centroid; instantaneous center of rotation; capacitance	n	Normal direction
d	Distance	\mathbf{N}	Normal component of reaction
$\mathbf{e}_n, \mathbf{e}_t$	Unit vectors along normal and tangent	O	Origin of coordinates
$\mathbf{e}_r, \mathbf{e}_\theta$	Unit vectors in radial and transverse directions	\mathbf{P}	Force; vector
e	Coefficient of restitution; base of natural logarithms	$\dot{\mathbf{P}}$	Rate of change of vector \mathbf{P} with respect to frame of fixed orientation
E	Total mechanical energy; voltage	q	Mass rate of flow; electric charge
f	Scalar function	\mathbf{Q}	Force; vector
f_f	Frequency of forced vibration	$\dot{\mathbf{Q}}$	Rate of change of vector \mathbf{Q} with respect to frame of fixed orientation
f_n	Natural frequency	$(\dot{\mathbf{Q}})_{Oxyz}$	Rate of change of vector \mathbf{Q} with respect to frame $Oxyz$
\mathbf{F}	Force; friction force	\mathbf{r}	Position vector
g	Acceleration of gravity	$\mathbf{r}_{B/A}$	Position vector of B relative to A
G	Center of gravity; mass center; constant of gravitation	r	Radius; distance; polar coordinate
h	Angular momentum per unit mass	\mathbf{R}	Resultant force; resultant vector; reaction
\mathbf{H}_O	Angular momentum about point O	R	Radius of earth; resistance
$\dot{\mathbf{H}}_G$	Rate of change of angular momentum \mathbf{H}_G with respect to frame of fixed orientation	\mathbf{s}	Position vector
$(\dot{\mathbf{H}}_G)_{Gxyz}$	Rate of change of angular momentum \mathbf{H}_G with respect to rotating frame $Gxyz$	s	Length of arc
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Unit vectors along coordinate axes	t	Time; thickness; tangential direction
i	Current	\mathbf{T}	Force
I, I_x, \dots	Moments of inertia	T	Tension; kinetic energy
\bar{I}	Centroidal moment of inertia	\mathbf{u}	Velocity
I_{xy}, \dots	Products of inertia	u	Variable
J	Polar moment of inertia	U	Work
k	Spring constant	U_{1-2}^{NC}	work done by non-conservative forces
k_x, k_y, k_O	Radii of gyration	\mathbf{v}, v	Velocity
		v	Speed
		$\bar{\mathbf{v}}, \bar{v}$	Velocity of mass center
		$\mathbf{v}_{B/A}$	Velocity of B relative to frame in translation with A
		$\mathbf{v}_{P/\mathcal{F}}$	Velocity of P relative to rotating frame \mathcal{F}

V	Vector product	θ	Angular coordinate; Eulerian angle; angle; polar coordinate
V	Volume; potential energy	μ	Coefficient of friction
w	Load per unit length	ρ	Density; radius of curvature
W, W	Weight; load	τ	Periodic time
x, y, z	Rectangular coordinates; distances	τ_n	Period of free vibration
$\dot{x}, \dot{y}, \dot{z}$	Time derivatives of coordinates x, y, z	ϕ	Angle of friction; Eulerian angle; phase angle; angle
$\bar{x}, \bar{y}, \bar{z}$	Rectangular coordinates of centroid, center of gravity, or mass center	φ	Phase difference
α, α	Angular acceleration	ψ	Eulerian angle
α, β, γ	Angles	ω, ω	Angular velocity
γ	Specific weight	ω_f	Circular frequency of forced vibration
δ	Elongation	ω_n	Natural circular frequency
ε	Eccentricity of conic section or of orbit	Ω	Angular velocity of frame of reference
λ	Unit vector along a line		
η	Efficiency		